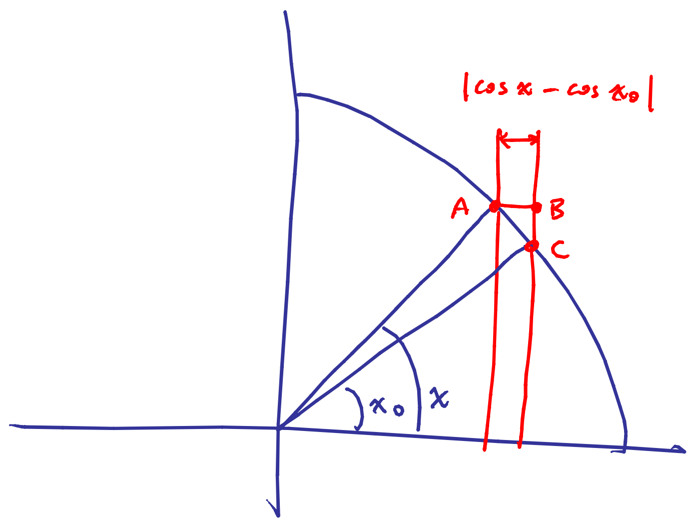
Hw 6 sol's

Note Title 3/30/2009

11.1 (2)

Picture:




Shortest line segment between vertical lines is the line segment $\overline{AB}$

In particular,

$$\left|\cos(x) - \cos(x_0)\right| = \text{length}(\overline{AB}) \leq \text{arclength}(\overset{\frown}{AC})$$

$$\text{arclength}(\overset{\frown}{AC}) = |x - x_0|$$

Take $\varepsilon > 0$. Choose $\delta = \varepsilon$

If $|x - x_0| < \delta \implies |\cos(x) - \cos(x_0)| \leq |x - x_0| < \delta = \varepsilon$

Therefore, $\cos x$ is continuous at $x_0$.

11.1 (5) Take $\varepsilon = 1$

By continuity, $f(x) \underset{1}{\approx} f(x_0)$ for $x \approx x_0$

Therefore:

$$f(x_0) - 1 < f(x) < f(x_0) + 1 \quad \text{for } x \approx x_0$$

$\Rightarrow$ $f(x)$ bounded for $x \approx x_0$

$\Rightarrow$ $f(x)$ locally bounded at $x_0$.

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11.2 (1)

Let $\varepsilon > 0$. Choose $\delta = \min\left\{1, \frac{\varepsilon}{2}\right\}$

Then if $|x - 0| < \delta$

$$\left| \frac{1-x}{x^2+1} - 1 \right| = \left| \frac{-x-x^2}{x^2+1} \right| = \frac{|x||1+x|}{|x^2+1|} \leq |x||1+x| \underset{|x|<1}{\leq} 2|x| < 2\delta \leq \varepsilon$$

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