

# Topic Summary - 18.100A

Note Title

5/16/2007

## Completeness of Real #'s

- Limit of an increasing sequence
- Bounded increasing sequences have limits

## Limit of a sequence

- Definition + what does  $\lim a_n = \infty$  mean?
- Theorems

- $\epsilon$ - $\delta$  principle
- Limit comparison / squeeze
- Limit location
- sequence location
- subsequences
- sums, products, quotients

- Bolzano - Weierstrass
- Cauchy sequences
- sup, inf
  - completeness prop for sets

## Series (of numbers)

- Def of convergence / absolute convergence

- Convergence tests
  - $n^{\text{th}}$  term test
  - comparison test for POSITIVE series
  - tail convergence
  - integral test
  - ratio test
  - $n^{\text{th}}$  term test
  - asymptotic comparison
  - Alternating series
- Power series
  - Radius of convergence
  - adding, multiplying

## Continuity

- Definition
- limits of functions
  - relationship to limits of sequences
  - sums, products, quotients
  - limit location
  - function location

- Squeeze thm
- comparison thm
- IVT
- Compact intervals + continuity
  - sequential compactness
  - boundedness of cont functions on cpt intervals
  - extremal points
  - Uniform continuity

## Differentiation

- Definition
- sums, products, chain rule, inverse function thm
- Local properties
  - locally decreasing/increasing
  - local min/max
- MVT
  - increasing, decreasing
  - constant functions
- Taylor approximation

## Riemann integral

- Def of integrability
- continuous functions on cpt intervals are integrable
- monotone functions on cpt intervals are integrable
- Def of integral
- Riemann sums
- properties of integrals
- interval addition
- 1<sup>st</sup> + 2<sup>nd</sup> fund thm of calc
- improper integrals

## Sequences + series of functions

- pointwise / uniform convergence
- Elementary criterion for uniform convergence
- Weierstrass M-test

- term-by-term integration
- term-by-term differentiation
- power series
  - integrating
  - differentiating