

18.100A: EXAM 1

You may use your book, but nothing else. Cite theorems you use either by their name (e.g. “Cauchy’s criterion for convergence”) or by their number in the book (e.g. “Theorem 8.4B”).

1. Prove that

$$\lim_{n \rightarrow \infty} 1 + \frac{1}{n^2} = 1$$

directly from the definition of a limit.

2. Show that a bounded sequence must have a cluster point.

3. Show that the following series converge or diverge:

$$(a) \quad \sum (-1)^n \frac{n+1}{n^2}$$

$$(b) \quad \sum \left(1 + \frac{1}{n^2}\right)$$

$$(c) \quad \sum \left(\frac{\sin n}{2}\right)^n$$

4. Suppose that A is a subset of \mathbb{R} and $\sup A = M$. Show that if $L < M$, then there is an element $a \in A$ such that $L < a$.

5. Determine the radius of convergence of

$$\sum \frac{x^n}{(2n)!}.$$