

18.100A Practice Midterm I.

This exam is too long. Try to do as many problems as you can, the ones you feel most comfortable with first. They are not in the order of difficulty. Each problem counts 10 points.

You can use the book but no notes, problem sets, calculators, etc.
Cite theorems used, by name or number.

1. Prove $\lim_{n \rightarrow \infty} \frac{3n^2 + 1}{n^2 + 1} = 3$ directly from the definition of the limit.

2. Let S be the set $S = \left\{ (-1)^n \left(1 + \frac{(-1)^n}{n} \right) : n = 1, 2, 3, \dots \right\}$.

Find the max, min, sup, inf, and cluster points, of S and for each cluster point a sequence of elements of S converging to it.

3. Find the radius of convergence R for the series $\sum_1^{\infty} \frac{x^n}{2^{3n} \sqrt{n}}$, and determine (with proof) whether it converges at the two endpoints $x = \pm R$.

4. Let $a_n = \frac{c^n}{n!}$, where $c > 1$. Prove a_n is monotone for $n \gg 1$.

5. Let L and M be two numbers such that, given $\epsilon > 0$, $L \approx_{\epsilon} M$. Prove that $L = M$. Use contraposition.

6. Let a_n be an increasing sequence, and suppose that $\max\{a_n : n = 1, 2, 3, \dots\}$ exists. Draw a conclusion about the sequence and prove it.

7. If $\{x_n\}$ is a sequence of points in the closed interval $[a, b]$, then it has a subsequence which converges to a point c , where c is also in the interval. Prove it.

8. a) Prove that if $\sum a_n$ is absolutely convergent, then $\sum a_n^2$ is convergent.
b) Show by counterexample that the word “absolutely” can not be dropped in part (a).

9. Let S be a non-empty set of real numbers bounded above, and let $\bar{m} = \sup S$. By considering **inf-1** and **inf-2**, prove that $\inf\{\bar{m} - x : x \in S\} = 0$.