HOMEWORK 3

THIS ASSIGNMENT IS DUE TUESDAY, FEBRUARY 25

Turn in problems 1 and 3

1. Show that if $A \hookrightarrow X$ is a cofibration, then $A \times Y \hookrightarrow X \times Y$ is a cofibration.

2. Suppose that $A \hookrightarrow X$ is a cofibration. Show that the inclusion

$$X \times S^{n-1} \cup_{A \times S^{n-1}} A \times D^n \hookrightarrow X \times D^n$$

is a cofibration.

Hint: There is a sneaky trick which makes this problem almost trivial: there is a bijective correspondence

$$\begin{array}{ccc} X \times S^{n-1} \cup A \times D^n & \longrightarrow & \underline{\mathrm{Map}}(I, Z) \\ \downarrow & \nearrow & \downarrow{\scriptstyle ev_0} \\ X \times D^n & \longrightarrow & Z \end{array} \quad \leftrightarrow \quad \begin{array}{ccc} A & \longrightarrow & \underline{\mathrm{Map}}(D^n \times I, Z) \\ \downarrow & \nearrow & \downarrow \\ X & \longrightarrow & \underline{\mathrm{Map}}(S^{n-1} \times I \cup D^n \times \{0\}, Z) \end{array}$$

3. Suppose that $A \hookrightarrow X$ is a cofibration.
(a) Show that the canonical map $\mathrm{Cone}(i) \to X/A$ is a homotopy equivalence. Here $\mathrm{Cone}(i)$ is the unreduced mapping cone.
(b) Deduce that there is an isomorphism $H^*(X, A) \cong \widetilde{H}^*(X/A)$.

4. Show that if X is well pointed, then the quotient map

$$\mathrm{Susp}(X) \to \Sigma X$$

is a homotopy equivalence. (I found problems 1,2 and 3a helpful, but they might be completely unnecessary.)