

## HOMEWORK 5

DUE: TUESDAY, MARCH 11

*Turn in Problems 1 and 2*

1. (Hatcher) Consider the equivalence relation  $\sim_w$  generated by weak homotopy equivalence:  $X \sim_w Y$  if there are spaces  $X = X_1, X_2, \dots, X_n = Y$  with weak homotopy equivalences  $X_i \rightarrow X_{i+1}$  or  $X_i \leftarrow X_{i+1}$  for each  $i$ . Show that  $X \sim_w Y$  iff  $X$  and  $Y$  have a common CW-approximation.

2. (Hatcher, Sec 4.2, problem 15) Show that a closed simply-connected 3-manifold is homotopy equivalent to  $S^3$ . [Use Poincaré duality, and also the fact that closed manifolds are homotopy equivalent to CW-complexes, from Corollary A.12 in the appendix of Hatcher. The stronger statement that a closed simply connected 3-manifold is homeomorphic to  $S^3$  is proven. This is the Poincaré conjecture.]

3. Argue that there exists a map  $\alpha : S^2 \rightarrow S^2 \vee S^1$  so that the inclusion

$$S^1 \hookrightarrow (S^2 \vee S^1) \cup_{\alpha} D^3$$

induces an isomorphism on  $\pi_1$  and  $\tilde{H}_*$ , but is not a homotopy equivalence.