

16 - Examples of SSS

Note Title

4/6/2010

Examples:

$$E = F \times B$$
$$\downarrow$$
$$B$$

Spatial series collapses ($d_r = 0$ $r \geq 2$)

$$H_s(B; H_t(F)) \Rightarrow H_{s+t}(B \times F)$$

Universal coefficient

$$\text{Cor } H_n(B \times F) = \bigoplus_{s+t=n} H_s(B; H_t(F))$$

$$0 \rightarrow H_s(B) \otimes H_t(F) \rightarrow H_s(B; H_t(F)) \rightarrow \text{Tor}_1(H_{s-1}(B), H_t(F))$$

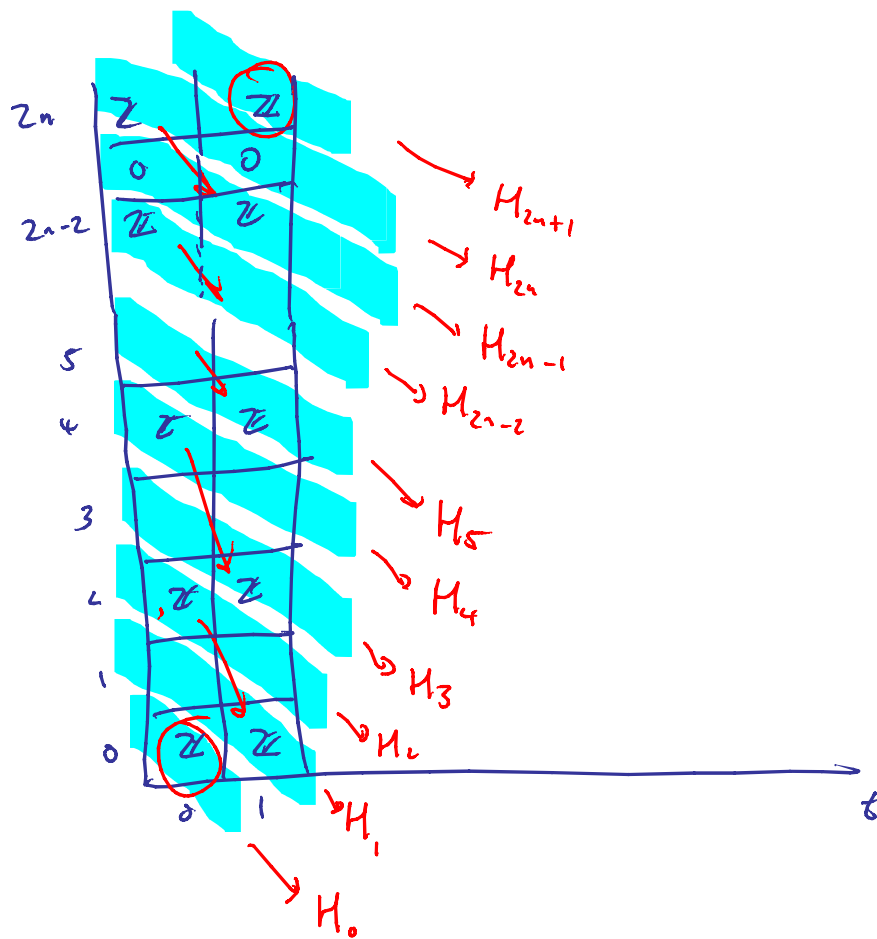
\downarrow
0

"Kunz's theorem"

Example:

$$S^1 \rightarrow S^{2n+1} \rightarrow \mathbb{C}P^n$$

$$H_t(\mathbb{C}P^n; H_t(S^1)) \cong H_t(\mathbb{C}P^n) \otimes H_t(S^1)$$



Example

$$\mathbb{Z}/2 \rightarrow S^\infty \rightarrow \mathbb{RP}^\infty$$

$$H_0(F) = \mathbb{Z} \oplus \mathbb{Z} \cong \mathbb{Z}[\pi_1]$$



π_1 introduces free

$$H_s(\mathbb{RP}^\infty, H_+(F)) \begin{cases} \text{1 free spiral summand} \\ \text{for } s > 0 \end{cases}$$

\Rightarrow collapses

$$H_s(\mathbb{RP}^\infty, H_0(F)) \cong H_s(S^\infty)$$

\cong

$$H_s(C_*^{\text{cell}}(\widetilde{\mathbb{RP}^\infty}) \otimes_{\mathbb{Z}[\pi_1]} \mathbb{Z}[\pi_1])$$

\cong

$$H_s(C_*^{\text{cell}}(S^\infty))$$

✓
possible by
twisted
coeff's

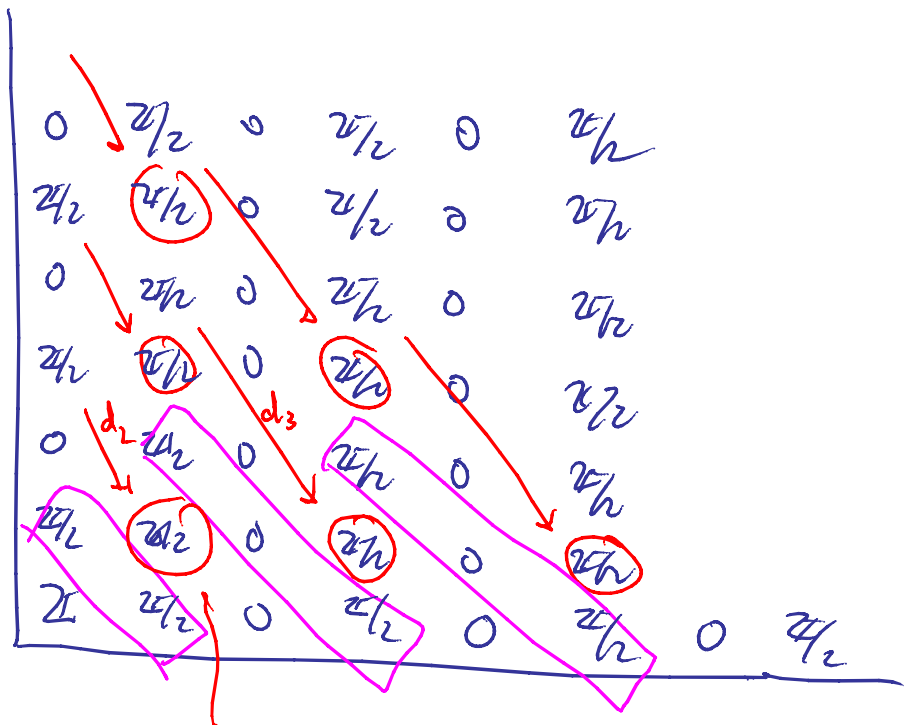
Other examples:

$$H_+(S^{n+1})$$

Holler extension:

$$K(\mathbb{Z}/2, 1) \rightarrow K(\mathbb{Z}/4, 1) \rightarrow K(\mathbb{Z}/2, 1)$$

$$\left. \begin{array}{c} \phantom{K(\mathbb{Z}/2, 1)} \\ \phantom{K(\mathbb{Z}/4, 1)} \end{array} \right\} H_2 = \mathbb{Z} \oplus \mathbb{Z}/4 \oplus 0 \oplus \mathbb{Z}/2 \oplus 0 \oplus \mathbb{Z}/2$$



most die

$$E_4 = E_\infty \quad \mathbb{Z}/4, \quad \text{Not } \mathbb{Z}/2 \oplus \mathbb{Z}/2$$

$$0 \rightarrow \mathbb{Z}/2 \rightarrow H_{2+1}(K(\mathbb{Z}/2, 1)) \rightarrow \mathbb{Z}/2 \rightarrow 0$$