# HOMEWORK 10

DUE: THURSDAY, 4/22/10 (TUESDAY IS PATRIOT'S DAY)

1. Inductively using the Serre spectral sequences for the fiber sequences
$$U(n-1) \to U(n) \to S^{2n-1},$$
show that there is an isomorphism
$$H^*(U(n)) \cong \Lambda[e_1, e_3, e_5, \dots, e_{2n-1}]$$
(an exterior algebra on generators in degrees $2i-1$).

2. Suppose that
$$\alpha : G \to G$$
is an inner automorphism (conjugation by an element of $G$). Show that the induced map
$$\alpha_* : BG \to BG$$
is homotopic to the identity.

3. Show that for complex line bundles $L_i$ over a space $X$, $c_1(L_1 \otimes L_2) = c_1(L_1) + c_1(L_2)$. (Hint - consider the "universal example": the external tensor product $L_{univ} \otimes L_{univ}$ over $\mathbb{C}P^\infty \times \mathbb{C}P^\infty$).

4. Viewing $\mathbb{C}P^\infty$ as the space of complex lines in $\mathbb{C}^\infty$, consider the line bundle $\mathcal{L}$ over $\mathbb{C}P^\infty$ whose fiber over a line $L \in \mathbb{C}P^\infty$ *is* the line $L$. Show that this bundle is isomorphic to the universal line bundle. (Hint: use the quotient map
$$S^\infty \to \mathbb{C}P^\infty$$
to give a model for $EU(1) \to BU(1)$. The universal vector bundle was defined to be $EU(1) \times_{U(1)} \mathbb{C} \to BU(1)$.)