HOMEWORK 9

DUE: TUESDAY, 4/13/06

In the following problems, when I say "compute the Serre spectral sequence", what I mean is

(1) Identify E^2.
(2) Compute the differentials.
(3) Analyze E^∞ and its relationship to the (co)homology of the total space.

1. Let $S^1 \to S^3 \to S^2$ be the Hopf fibration. Compute the Serre spectral sequence for this fibration.

2. (Hatcher's spectral sequence notes, Sec. 1.1, prob. 1) Let $\phi_n : S^k \to S^k$ be the degree n map for $n, k > 1$. Compute the homology of the homotopy fiber of ϕ_n.

3. (Hatcher's spectral sequence notes, Section 1.2, problem 1) Use the Serre spectral sequence to compute $H^*(F; \mathbb{Z})$ for F the homotopy fiber of a map $S^k \to S^k$ of degree n for $k, n > 1$, and show that the cup product structure in $H^*(F; \mathbb{Z})$ is trivial.

4. Pretend that you don't know that $K(\mathbb{Z}, 2) = \mathbb{C}P^\infty$. Give a new computation of $H^*(K(\mathbb{Z}, 2))$ with its cup product structure by applying the cohomological Serre spectral sequence to the homotopy fiber sequence

$$K(\mathbb{Z}, 1) \to * \to K(\mathbb{Z}, 2).$$