

B - Morava Change of Rings

Note Title

11/12/2008

Chromatic layers $\longleftrightarrow \text{Ext}_{\mathbb{B}\mathbb{P}_2/\mathbb{B}\mathbb{P}}(\mathbb{B}\mathbb{P}_2 / (p^\infty, \dots, v_{n-1}^\infty) [v_n^{-1}])$

Compute this?

Inductive computation of $\text{Ext}_{\mathbb{B}\mathbb{P}_2/\mathbb{B}\mathbb{P}}(\mathbb{B}\mathbb{P}_2 / (p, \dots, v_m, v_{m+1}^\infty, \dots, v_n^\infty) [v_n^{-1}])$

$$\left(\mathbb{B}\mathbb{P}_2 / I_m \right) / (v_m^\infty, \dots, v_{n-1}^\infty) [v_n^{-1}]$$

||

$$\text{colim}_K \left(\mathbb{B}\mathbb{P}_2 / I_m \right) / (v_m^K, v_{m+1}^\infty, \dots) [v_n^{-1}]$$

Get a BSS

$$\text{Ext} \left(\left(\mathbb{B}\mathbb{P}_2 / I_{m+1} \right) / (v_{m+1}^\infty, \dots, v_{n-1}^\infty) [v_n^{-1}] \right) \left[\frac{1}{v_m^K} \right]$$

$$\Rightarrow \text{Ext} \left(\left(\mathbb{B}\mathbb{P}_2 / I_m \right) / (v_m^\infty, \dots, v_{n-1}^\infty) [v_n^{-1}] \right)$$

diffs

$$x \in C^s \left(\frac{BP_n}{(V_{n+1}^{i_{n+1}}, \dots, V_{n-1}^{i_{n-1}})} [V_n^{-1}] \right)$$

$$\left[\frac{x}{\begin{matrix} i_{n+1} & \dots & i_{n-1} \\ V_{n+1} & \dots & V_{n-1} \end{matrix}} \right] \in \text{Ext}_{BP_n}^s \left(\frac{BP_n}{I_{n+1}} \middle/ \frac{[V_n^{-1}]}{(V_{n+1}^{\infty}, \dots, V_{n-1}^{\infty})} \right)$$

$$d(x) \equiv 0 \quad (p, v_1, \dots, v_m, V_{n+1}^{i_{n+1}}, \dots, V_{n-1}^{i_{n-1}})$$

I- BSS

$$d \left[\frac{x}{\begin{matrix} i_{n+1} & \dots & i_{n-1} \\ V_m^k & V_{n+1} & \dots & V_{n-1} \end{matrix}} \right] = \frac{y}{V_m^k V_{n+1}^{i_{n+1}} \dots V_{n-1}^{i_{n-1}}}$$

$$\iff dx = v_m^r y \quad (p, v_1, \dots, v_m, v_{n+1}^{i_{n+1}}, \dots, v_{n-1}^{i_{n-1}})!$$

$$\text{Ext}_{BP_n} \left(\frac{BP_n}{I_n} [V_n^{-1}] \right) \Rightarrow \dots \Rightarrow \text{Ext}_{(p, \dots, V_{n+1}^{\infty})} \left(\frac{BP_n}{(p, \dots, V_{n+1}^{\infty})} [V_n^{-1}] \right)$$

$\underbrace{\hspace{10em}}_{K(n)} \quad \begin{matrix} V_{n-1} \text{ BSS} & & V_0 \text{ BSS} \end{matrix}$

Morava Change of rings thm

$$\Sigma_i(n) = \text{Hopf algebra} / K(n) \quad \text{“Morava stable algebra”}$$

$$\mathcal{J}_n = \text{certain sp} \quad \text{“Morava stable sp”}$$

$$S(n) = \Sigma_i(n) \otimes_{K(n)} \mathbb{F}_p \quad v_n \mapsto 1$$

Change of Rings:

$$\text{Ext}_{\text{BR}_{\mathbb{F}_p}}(K(n)_n) \stackrel{(1)}{\cong} \text{Ext}_{S(n)}(\mathbb{F}_p, \mathbb{F}_p)[v_n^{\pm 1}]$$

\parallel (2) $\text{Ext}_{\Sigma(n)}^u(K(n), K(n))$

$$H_c^*(S_n, \mathbb{F}_p)[v_n^{\pm 1}]$$

$$\Sigma(n) = E(n)_n \underset{(p, v_1, \dots, v_{n-1})}{E(n)} = K(n)_n E(n)$$

So (1) \leftrightarrow algebraic telescope u_j :

$$\text{Ext}_{E(n)_n E(n)}(E(n)_n, E(n)_n / I_n) \cong \text{Ext}_{\text{BR}_{\mathbb{F}_p}}(\text{BR}_{\mathbb{F}_p}, \text{BR}_{\mathbb{F}_p} / I_n[v_n^{\pm 1}])$$

\parallel

$$\text{Ext}_{E(n)_n E(n)} / I_n (E(n)_n / I_n, E(n)_n / I_n)$$

ANS for $v(n-1) E(n)$

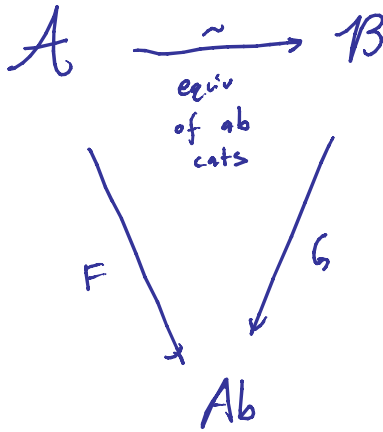
ANS for $v_n^{-1} v(n-1)$

We will sketch

$$\text{Ext}_{\text{BR}_{\mathbb{F}_p}}(\text{BR}_{\mathbb{F}_p} / I_n[v_n^{\pm 1}]) \stackrel{(2)}{=} H_c^*(S_n, \mathbb{F}_p)[v_n^{\pm 1}] \stackrel{(3)}{\cong} \text{Ext}_{S(n)}(\mathbb{F}_p)[v_n^{\pm 1}]$$

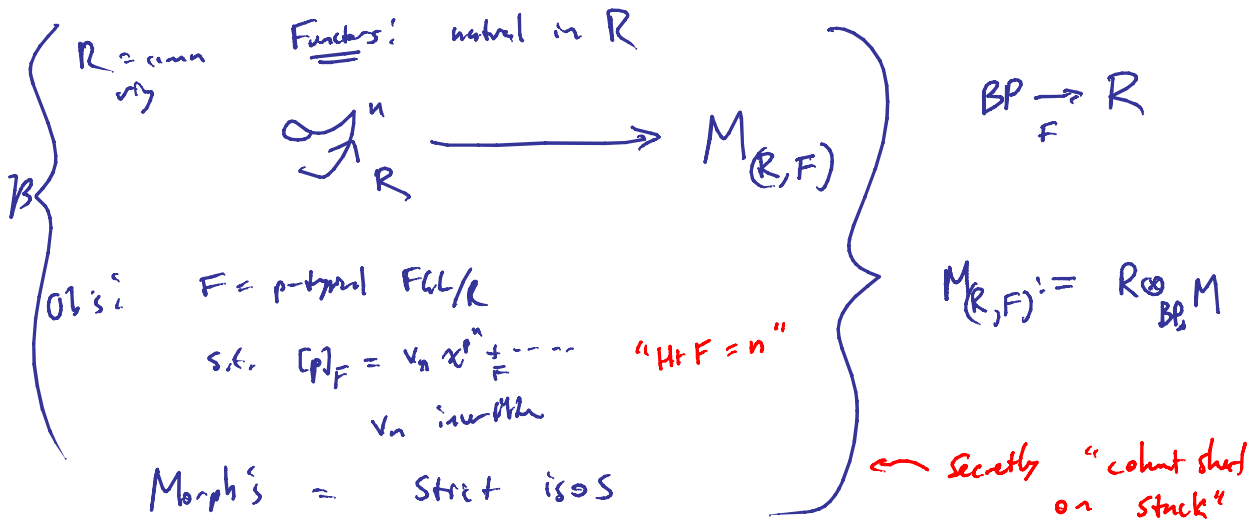
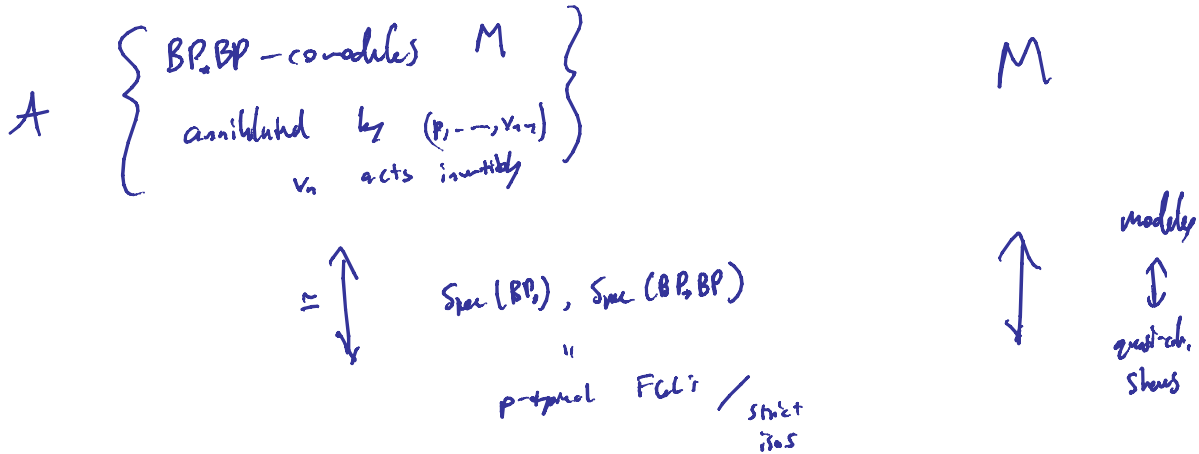
Idea:

kehd. (2)



$$\Rightarrow R^i F \cong R^i G$$

[Warning: finiteness assumptions omitted]



$\approx \downarrow (\neq)$

$$(M_{\mathbb{F}_n} \otimes_{\mathbb{F}_p})_{H \neq n}$$

$$e \left\{ S_n \times \text{Gal-modules} / \overline{\mathbb{F}_p} \quad M_0 \right\}$$

$$M_0 = M_{(\overline{\mathbb{F}_p}, \mathbb{F}_n)}$$

$$F_n = p\text{-typical formal gp} / \overline{\mathbb{F}_p}$$

$$[p]_{F_n} = x^{p^n}$$

$$S_n = \text{Aut}^{\text{strict}}(F_n)$$

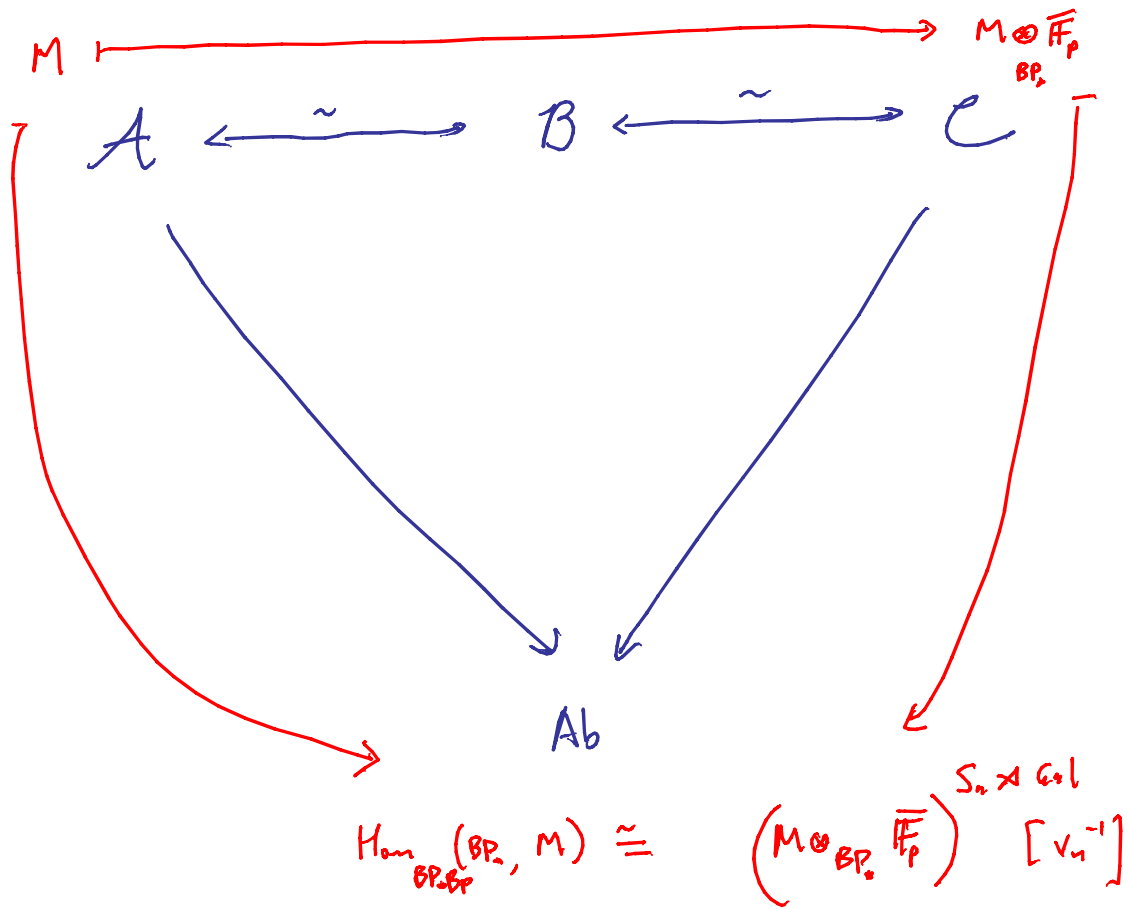
$$\text{Gal} = \text{Gal}(\overline{\mathbb{F}_p} / \mathbb{F}_p)$$

(*) follows from

(1) Thus F_n 's / separably closed fields of char p classified by Height

\Rightarrow compared $\mathcal{F}_k^n \cong \mathbb{F}_k^* \curvearrowright \text{Aut}(F)$
1 object

(2) Sheaf is determined by "Stalks at $\overline{\mathbb{F}_p}$ points"



Get

$$\text{Ext}_{BP, BP}(BP, M) \cong H_c^1(S_n, M \otimes_{BP} \overline{\mathbb{F}}_p)^{\text{Gal}}$$

In particular:

$$\begin{aligned} \text{Ext}_{BP, BP}(BP, BP/I_n[v_n^{-1}]) &\cong H_c^1(S_n, \overline{\mathbb{F}}_p)^{\text{Gal}} [v_n^{-1}] \\ &\quad \uparrow \text{trivial } S_n\text{-mod} \\ &\cong H^1(S_n, \mathbb{F}_p) [v_n^{-1}] \end{aligned}$$

$$(3) \quad H^1(S_n, \mathbb{F}_p) \cong \text{Ext}_{S^{(n)}}(\mathbb{F}_p)$$

$$H_c^1(S_n, \overline{\mathbb{F}}_p) \cong \text{Ext}_{S^{(n)}}(\overline{\mathbb{F}}_p) + \text{"Galois descent"}$$

$$\Sigma_c^1(n) = K(n) \otimes E(n)$$

$$E(n) \otimes E(n) = E(n)_2 \otimes_{BR} BR \otimes_{BR} E(n)_2$$

↙
"ladder
crosses"

$$= \frac{E(n)_2 [t_1, t_2, \dots]}{(n_R(x_{n+1}), n_R(x_{n+2}), \dots)}$$

$$K(n) \otimes E(n) = \frac{K(n)_2 [t_1, \dots]}{(n_R(x_{n+1}), \dots)}$$

lem!

$$\Sigma_c^1(n) \cong K(n) \otimes_{BR} BR \otimes_{BR} K(n)_2$$

idem!

$$\text{Rig}(K(n) \otimes E(n), R)$$



$$F \xrightarrow{\text{strut } 2} F'$$

$$F \xrightarrow{\text{has } k \text{ } \rightarrow} F' \text{ has } k'$$

Using inductive n_k formula

$$K(n), E(n) \cong \frac{\mathbb{F}_p\langle v_n \rangle [t_1, t_2, \dots]}{\left(v_n t_k^{p^n} = v_n^{p^k} t_k \right)_{k \geq 1}}$$

$$\cong S(n) \otimes_{K(n)} \mathbb{F}_p$$

(ungraded isomorphism)

$$\begin{aligned} K(n) &\longrightarrow \mathbb{F}_p \\ v_n &\longmapsto 1 \end{aligned}$$

$$S(n) \cong \frac{\mathbb{F}_p [t_1, t_2, \dots]}{\left(t_k^{p^n} = t_k \right)_{k \geq 1}} \quad \left(\text{actually } \mathbb{Z}/2(p^n-1)\text{-graded} \right)$$

Note

$$S_n = \text{Aut}^{\text{strict}}(F_n) \cong \text{Ring}(S(n), \overline{\mathbb{F}_p})$$

$$\cong \text{Ring}(S(n), \mathbb{F}_{p^n})$$

↗ $t_k^{p^n} = t_k \Rightarrow t_k \in \mathbb{F}_{p^n} \subset \overline{\mathbb{F}_p}$

$$\text{Ext}_{S(n) \otimes F_p} (F_p^n, F_p^n) \cong H_c^1(S_n, F_p^n)$$

$$\left[\begin{array}{l} \text{Compare } G = \text{SL}_2, G = \text{finite} \\ \text{Ext}_{k[G]}(k, k) \cong H^1(G, k) \end{array} \right]$$

Program

$$\begin{array}{c} \text{Ext}_{S(n)}(F_p, F_p)[v_1^{-1}] \cong \text{Ext}_{\text{BP, BP}}(\text{BP}_*/J_n[v_1^{-1}]) \\ \begin{array}{l} \uparrow \\ \text{use May SS} \end{array} \quad \begin{array}{l} \downarrow \\ \text{Main} \\ \text{diag} \\ \text{of } \rightarrow \end{array} \quad \begin{array}{l} \downarrow \\ \text{sequence BSS's} \\ \downarrow \\ \text{Ext}_{\text{BP, BP}}(\text{BP}_*/(v_1, \dots, v_{n-1})[v_1^{-1}]) \\ \downarrow \text{AWSS} \\ \pi_* M_n S \\ \downarrow \text{CSS} \\ \pi_* S \end{array} \end{array}$$

