<u>cohomology of $\Sigma(n)$</u>

$$\Sigma(n) = K(n)_*[t_1, \cdots] \big/ (v_n^{p^i} t_i = v_n t_i^{p^n})$$

<u>And</u>:

$$\mathrm{Ext}_{\Sigma(n)}(K(n)_*, K(n)_*) \otimes_{K(n)_*} \mathbb{F}_p \cong \mathrm{Ext}_{S(n)}(\mathbb{F}_p, \mathbb{F}_p)$$

$$S(n) = \mathbb{F}_p[t_1, \cdots] \big/ (t_i^{p^n} = t_i)$$

Filter $S(n)$ by powers of the unit coideal

$$E^0 S(n) = T(t_{i,j} \mid i > 0,\ j \in \mathbb{Z}/n)$$

Let $m = \lfloor pn/(p-1) \rfloor$

$$\Delta(t_{i,j}) = \begin{cases} \sum_{0 \le k \le i} t_{k,j} \otimes t_{i-k,k+j} & i < m \\ \sum'_{0 \le k \le i} t_{k,j} \otimes t_{i-k,k+j} + b_{i-n,j+n-1} & i = m \\ t_{i,j} \otimes 1 + 1 \otimes t_{i,j} + b_{i-n,j+n-1} & i > m \end{cases}$$

$$E_0 S(n)^* = V(P E_0 S(n)^*)$$

universal rest. env. alg.

x_{ij} dual to t_{ij}

$$[x_{i,j}, x_{k,l}] = \begin{cases} \delta^{l}_{i+j}\, x_{i+k,j} - \delta^{j}_{k+l}\, x_{i+k,l} & i+k \le m \\ 0 & o/w \end{cases}$$

$$\xi(x_{i,j}) = \begin{cases} 0 & i \le n/p-1 \\ -x_{i+n,\, j-n+1} & o/w \end{cases}$$

here $\delta^{i}_{j} = \begin{cases} 1, & i \equiv j \ (n) \\ 0, & o/w \end{cases}$

Let $L(n) = P E_0 S(n)^*$

Then $E^0 V(L(n)) = T(x_{ij})$

and there is a SS:

$$\mathrm{Ext}(E^0 V(L(n))) \Longrightarrow \mathrm{Ext}(V(L(n)))$$

$$\| $$

$$E(h_{ij}) \otimes P(b_{ij})$$

$$h_{ij} = [t_{ij}]$$

$$b_{ij} = \sum \tfrac{1}{p} \binom{p}{k} [t^{k}_{i,j} \mid t^{p-k}_{i,j}]$$

Now:

$$d\,t_{ij} = b_{i-n,\,j+n-1} \qquad \text{for} \quad i > m$$

So we have an acyclic tensor factor:

$$E(h_{ij} \mid i > m) \otimes \underbrace{P(b_{i-n,\,j+n-1} \mid i > m)}_{\begin{array}{c}\shortparallel \\ P(b_{k,j} \mid k > m-n)\end{array}}$$

So we obtain:

$$\begin{array}{l} L(n,m) \subseteq L(n) \\ \shortparallel \\ \langle x_{ij} \mid i \leq m \rangle \end{array}$$

$$\underbrace{\mathrm{Ext}\,(U(L(n,m)))}_{\begin{array}{c}\shortparallel \\ H^*\left(E(h_{ij} \mid i \leq m),\ d\,h_{ij} = \sum_{k+l=i} h_{k,j}\, h_{l,k+j}\right)\end{array}} \otimes P(b_{ij} \mid i \leq m-n) \Rightarrow \mathrm{Ext}\,(U(L(n)))$$

e.g.

$n=1$ $\qquad m = \left\lfloor \frac{p}{p-1} \right\rfloor = 1$

$p \geq 2$

$$E[h_{1,0}] \Rightarrow \mathrm{Ext}_{V(L(1))}(\mathbb{F}_p) \Rightarrow \mathrm{Ext}_{S(1)}(\mathbb{F}_p)$$

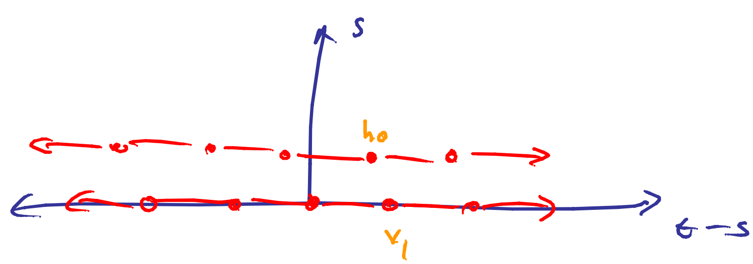
all must collapse

So $\quad \mathrm{Ext}_{S(1)}(\mathbb{F}_p) = E(h_{1,0})$

$$\Rightarrow \mathrm{Ext}_{BP_*BP}(BP_*, BP_*/p\,[v_1^{-1}]) \cong \Lambda_{\mathbb{F}_p}[h_0] \otimes \mathbb{F}_p[v_1, v_1^{-1}]$$

$$(s, t)$$
$$|v_1| = (0, 2(p-1))$$
$$|h_0| = (1, 2(p-1))$$

picture



$$\eta_R(v_1) \equiv v_1 + pt_1 \mod (p^p)$$

$$\Longrightarrow \quad \eta_R(v_1^k) = v_1^k + p^{i+1} m\, t_1 v_1^{k-1} + \cdots \mod p^{i+2}$$

$$k = p^i m \qquad p \nmid m$$

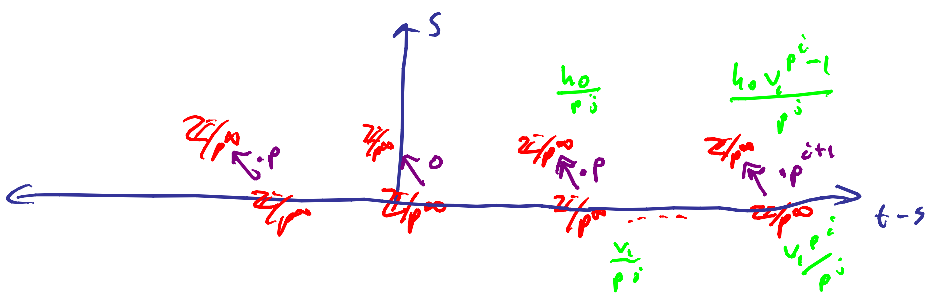
In <u>BSS</u>

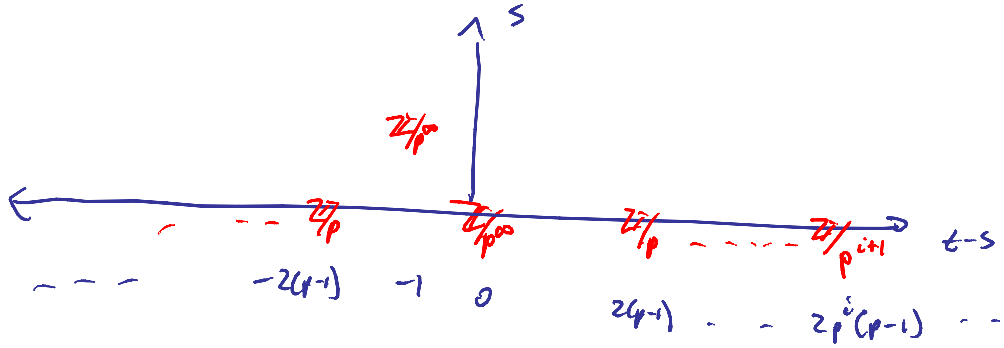
$$d\left(\frac{v_1^k}{p^j}\right) = \frac{v_1^{k-1} h_0}{p^{j-i-1}}$$

<u>So</u>:

$$\mathrm{Ext}^{s,*}_{BP_*BP}\left(BP_*/p^\infty[v_1^{-1}]\right) = \begin{cases} \left\langle \dfrac{v_1^{p^i m}}{p^{i+1}} \right\rangle \ (\mathbb{Z}/p^{i+1}), & * \neq 0,\ m \in \mathbb{Z},\ p \nmid m,\ i \geq 0,\ s = 0 \\ \mathbb{Z}/p^\infty, & s = 0,\ * = 0 \\ \mathbb{Z}/p^\infty, & s = 1,\ * = 0 \end{cases}$$

<u>Picture</u>:





$$\mathrm{Ext}^{s,t}_{BP_*BP}\left(BP_*/p^\infty[v_1^{-1}]\right) \Longrightarrow \mathrm{Ext}^{s,t}(M_1 S)$$

collapses

get
$$\pi_n M_1 S = \begin{cases} \mathbb{Z}/p^j\{x_{i/j}\} & n = 2i(p-1),\ \nu_p(i)=j-1 \\ \mathbb{Z}/p^\infty & n = 0, -1 \\ 0 & o/w \end{cases}$$

$$M_0 S = S_{E(0)} = S_{\mathbb{Q}}$$

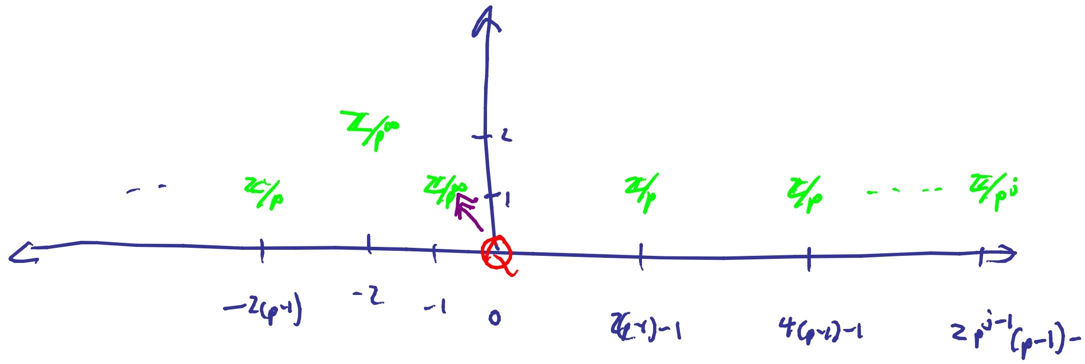
alg

Chromatic spectral sequence for $S_{E(1)}$:

$$\bigoplus_{n=0}^{1} \mathrm{Ext}^{s,t}_{BP_*BP}(M_n BP_*) \Longrightarrow \mathrm{Ext}^{s+n,t}_{BP_*BP}\left(\pi_* BP_{E(1)}\right)$$

$$\Downarrow$$

$$\pi_* S_{E(1)}$$



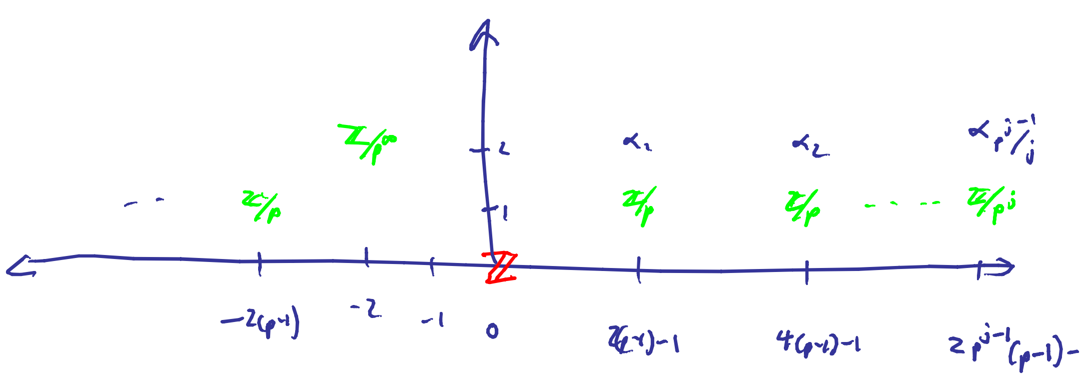
<ins>Diff'l</ins> d_1 in CSS

$$BP_* \longrightarrow (BP_*)_Q \longrightarrow (BP_*)/p^\infty$$

get

$$Ext^s(BP_*Q) \longrightarrow Ext^s(BP_*/p^\infty) \longrightarrow Ext^s(BP_*/p^\infty[v_1^{-1}])$$

$$\frac{1}{p^j} \longmapsto 1/p^j$$



$\mathbb{Z}$ and α_i $i \geq 0$ present in $\pi_* S$;

<u>2 constructions</u>: of $\alpha_{i/j}$:

(1) <u>Thm</u> $\Sigma^{2p^{j-1}(p-1)} M(p^j) \xrightarrow{v_1^{p^{j-1}}} M(p^j)$

$$\begin{tikzcd} S^{2sp^{j-1}(p-1)} \arrow[r] \arrow[rrr, bend right] & \Sigma^{2sp^{j-1}(p-1)-1} M(p^j) \arrow[r, "v_1^{sp^{j-1}}"'] & \Sigma^{-1} M(p^j) \arrow[r] & S^0 \end{tikzcd}$$

$\alpha_{sp^{j-1}/j}$ "greek letter construction"

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(2) <u>J hom</u>

$$O \xrightarrow{J} QS^0$$

$$\begin{tikzcd} & \pi_{2i(p-1)} S \arrow[r] & \pi_{2i(p-1)} S_{E(1)} \arrow[d, phantom, "\cong"] \\ \pi_{2i(p-1)} O = \mathbb{Z} \arrow[ur] \arrow[rr, two heads] & & \mathbb{Z}/p^j \{\alpha_{i/j}\} \end{tikzcd}$$

$j = \nu_p(i) + 1$

produce $\alpha_{i/j}$ classes for $\pi_* S$

To explain (2), we digress to talk about $S_{K(1)}$

$$\begin{tikzcd} M_1 S \arrow[r, equal] \arrow[d] & M_1 S \arrow[d] \\ S_{E(1)} \arrow[r] \arrow[d] & S_{K(1)} \arrow[d] \\ S_{\mathbb{Q}} \arrow[r] & S_{K(1),\mathbb{Q}} \end{tikzcd}$$

Morava Change of rings, $K(n)$-local version

F_n = Honda ht n formal grp $/\mathbb{F}_{p^n}$

$$[p]_{F_n} = x^{p^n}$$

$$\mathrm{Aut}(F_n) = \mathbb{S}_n \overset{p\text{-sylow}}{\supset} S_n = \mathrm{Aut}^{\text{strict}}(F_n)$$

$\uparrow$ all, including non-strict

$$\mathbb{G}_n = \mathbb{S}_n \rtimes \mathrm{Gal}(\mathbb{F}_{p^n}/\mathbb{F}_p)$$

$$\begin{tikzcd} W(\mathbb{F}_{p^n}) \arrow[r, phantom, "\subset"] & K \\ \mathbb{Z}_p \arrow[u, dash] \arrow[r, phantom, "\subset"] & \mathbb{Q}_p \arrow[u, dash, "n"'] \end{tikzcd}$$

E_n = Lubin-Tate coh. thy

$$\pi_* E_n = W(\mathbb{F}_{p^n})[[u_1, \dots, u_{n-1}]][u^{\pm}] \qquad |u_i| = 0$$

$$|u| = 2$$

$\tilde{F}_n = F_{E_n}$

$$[p]_{\tilde{F}_n}(x) = px +_{\tilde{F}_n} u^{p-1} u_1 x^p +_{\tilde{F}_n} \cdots +_{\tilde{F}_n} u^{p^{n-1}-1} u_{n-1} x^{p^{n-1}} +_{\tilde{F}_n} u^{p^n} x^{p^n}$$

"deformation of F_n"

$$\tilde{F}_n \Big|_{\mathrm{mod}\ (p, u_1, \dots, u_{n-2},\ u-1)} = F_n$$

$\mathbb{G}_n \circlearrowright \pi_* E_n$ "$\tilde{F}_n$ is a universal deformation of F_n"

Thm! Goerss–Hopkins–Miller

$\mathbb{G}_n \circlearrowright E_n$ E_∞ — E_∞-ring maps

$$H_c^*(\mathbb{G}_n, \pi_* E_n) \overset{\text{Morava}}{\cong} \mathrm{Ext}_{BP_*BP}\left(BP_* {}^{\wedge}_{I_n}[v_n^{-1}]\right)$$

$$\Downarrow \text{htpy fixed point SS} \qquad\qquad \Downarrow \text{ANSS}$$

$$\pi_* E_n^{h\mathbb{G}_n} \underset{\text{Dev. Hop.}}{\cong} \pi_* S_{K(n)}$$

"continuous htpy fixed points"

e.g. $n=1$

$$E_1 = K_p^\wedge \qquad F_1 = \hat{\mathbb{G}}_m / \mathbb{F}_p$$

$$\mathbb{S}_1 = \mathbb{G}_1 = \mathbb{Z}_p^\times \subset K_p^\wedge$$

$$\mathbb{G}_m \longrightarrow \mathbb{G}_m, \quad x \longmapsto x^k$$

$$[k]_*: K_p^\wedge \xrightarrow{\psi^k} K_p^\wedge \qquad \hat{\mathbb{G}}_m \xrightarrow[{[k]}]{\cong} \hat{\mathbb{G}}_m \quad \text{if } p \nmid k$$

Adams operator

$$\psi^k \qquad \begin{array}{ccc} \pi_{2n} K_p & \longrightarrow & \pi_{2n} K_p \\ \| & & \| \\ \mathbb{Z}_p & \xrightarrow{\cdot k^n} & \mathbb{Z}_p \end{array}$$

---

p odd $\Rightarrow$ $\mathbb{Z}_p^\times$ top'l cyclic

Thm: $\overline{\langle k \rangle} = \mathbb{Z}_p^\times$

fiber sequence

$$J = K_p^{h\mathbb{Z}_p^\times} \longrightarrow K_p \xrightarrow{\psi^k - 1} K_p$$

$$\| \\ S_{K(1)}$$

"J spectrum"

Adams Conj: cx. version

$$\begin{tikzcd}
BU[1/k] \arrow[r, "\psi^k - 1"] \arrow[rr, bend right, "\text{null}"'] & BU[1/k] \arrow[r] & BQS^0[1/k]
\end{tikzcd}$$

$$\Omega^\infty K = BU \times \mathbb{Z}$$

J-diagram:

$$\begin{tikzcd}
U_p \arrow[r] \arrow[dr, "J"'] & \Omega^\infty J \arrow[r] \arrow[d, dashed] & (BU \times \mathbb{Z})_p \arrow[r, "\psi^k - 1"] \arrow[d, dashed, "\text{Adams conj}"] & BU_p \arrow[d, equal] \arrow[dr, "J"] & \\
& QS^0_p \arrow[r] & QS^0/U_p \arrow[r] & BU_p \arrow[r] & BQS^0_p
\end{tikzcd}$$

Sullivan