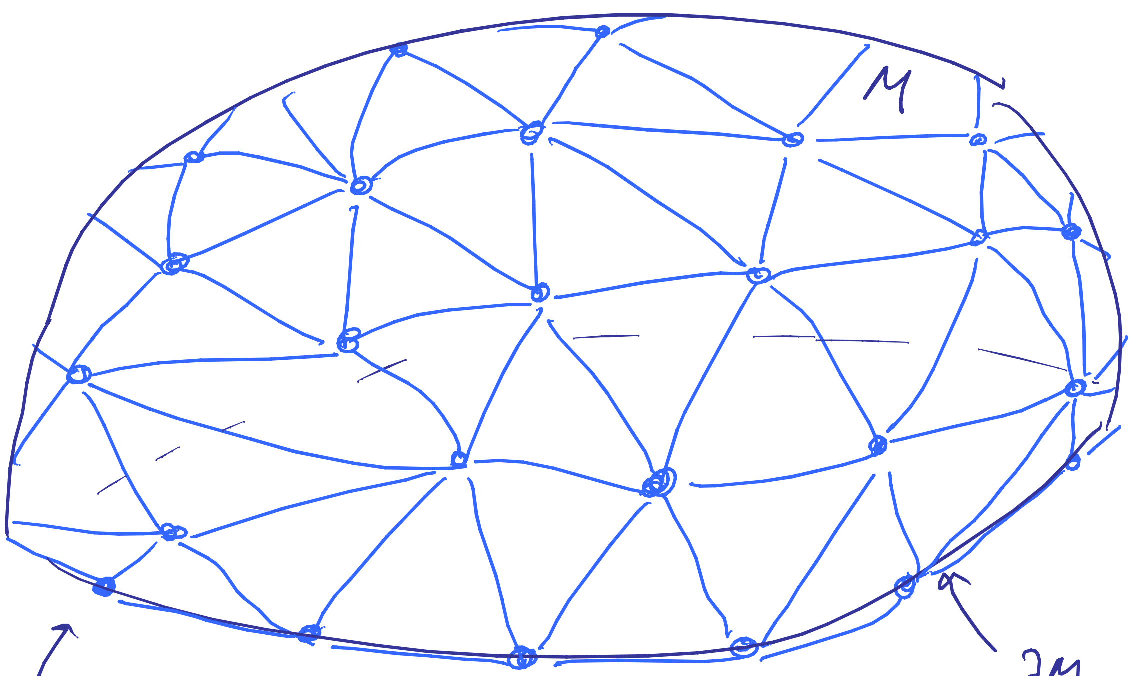
Euler Characteristic

Note Title 12/3/2009

Let M be a compact 2-manifold w/ boundary.

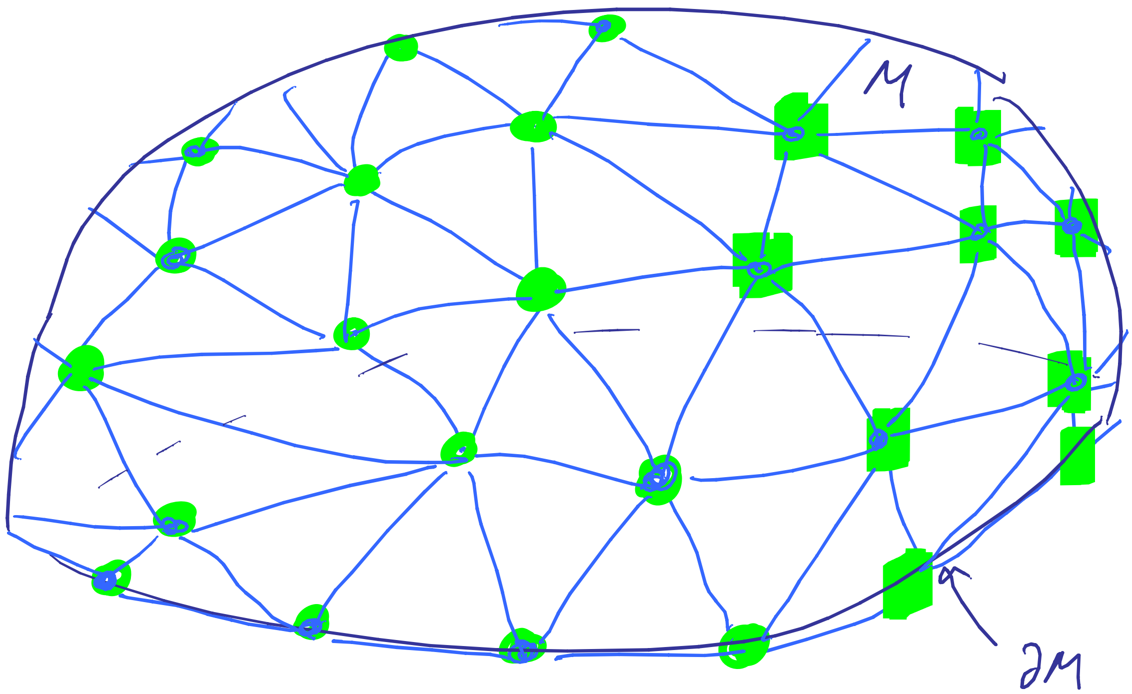
Give M a <u>triangulation</u>:

Subdivide the surface into triangles: (The edges of the triangles need not be straight)

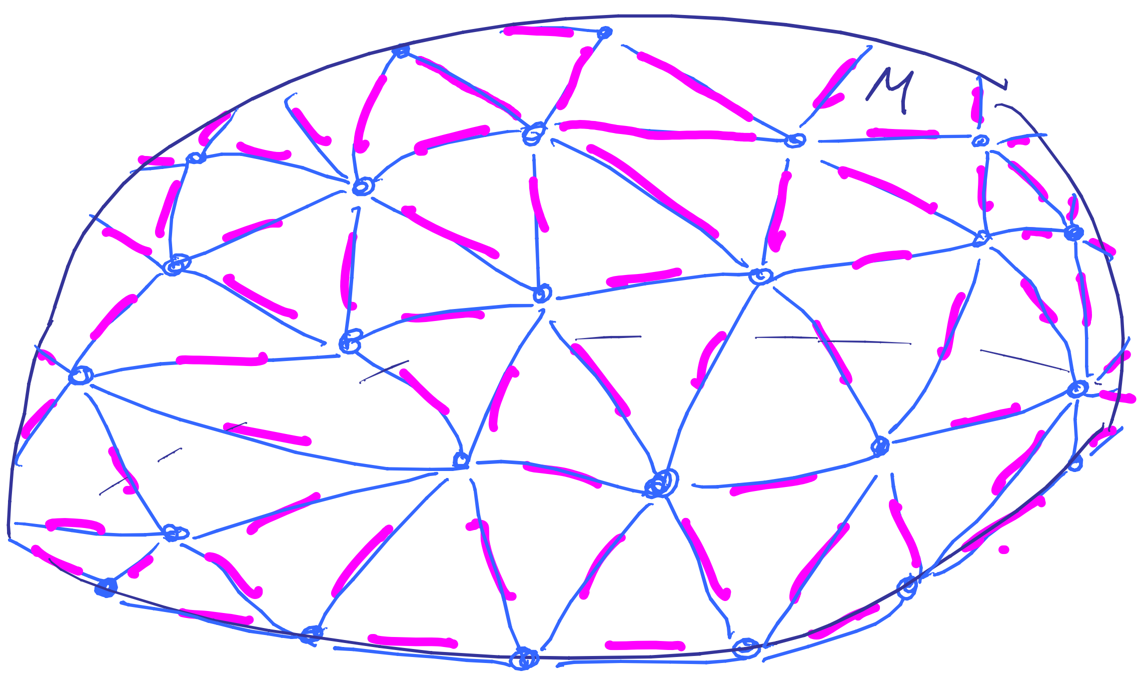


[Note: the triangulation is required to match up nicely on the boundary; so that the boundary is a union of edges.]

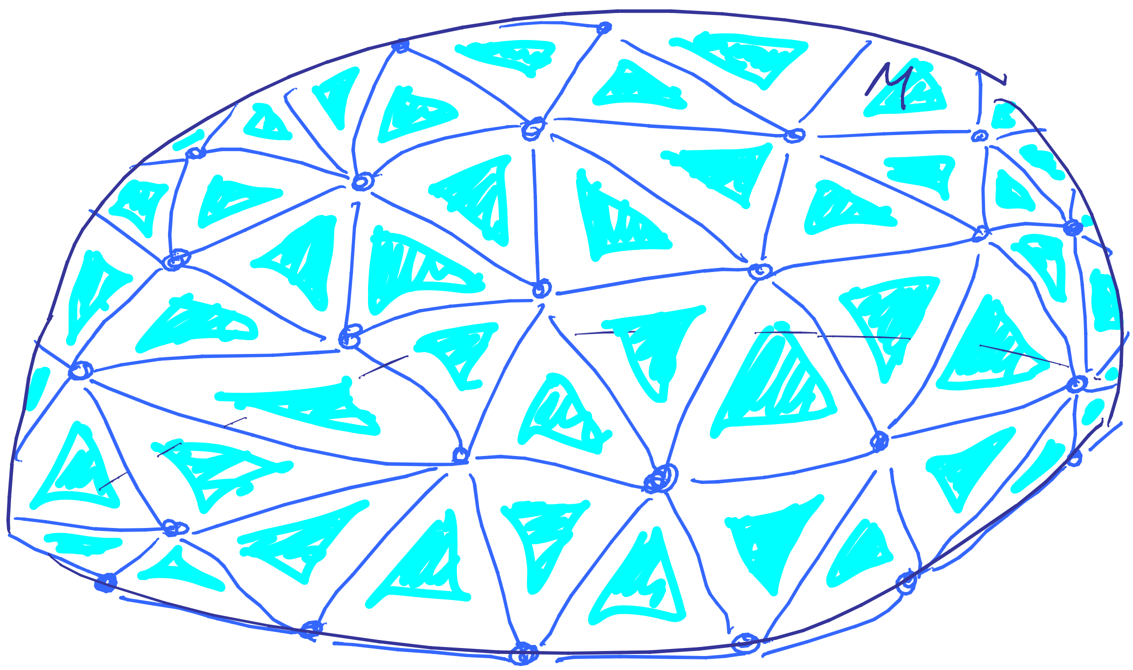
The triangulation has vertices:



The triangulation has edges:



and the triangulation has faces:



The Euler Characteristic $\chi(M)$

is defined to be

$$\chi(M) = F - E + V$$

where

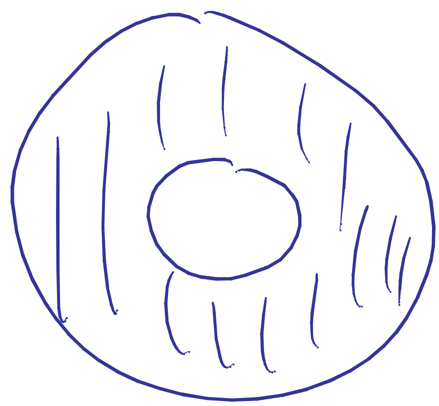
F = # faces

E = # edges

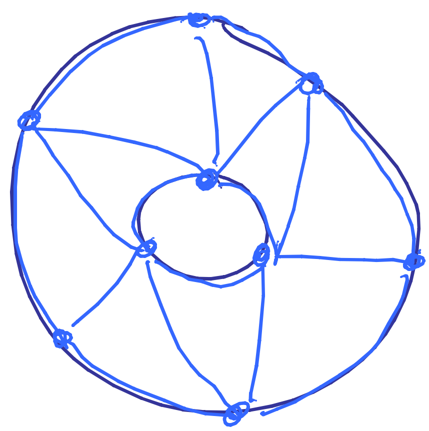
V = # vertices

Example consider

$M =$ annulus $= \{(x,y) \in \mathbb{R}^2 \mid 1 \le x^2+y^2 \le 2\}$



A triangulation:



$F = 9$

$E = 18$

$V = 9$

So $\chi(M) = 9 - 18 + 9$

$= 0$

Note that χ is <u>independent</u>

of triangulation.

(this is a consequence of the Gauss-Bonnet theorem, which expresses euler characteristic in terms of a quantity which is <u>independent</u> of triangulation)