18.950: PSET 3

1. (5 points) [Problem 23 from text] Let c be a Frenet curve in $\mathbb{R}^n$. Show that

$$\det(c', c'', c''', \ldots, c^{(n)}) = \prod_{i=1}^{n-1} \kappa_i^{n-i}.$$

[Hint: write everything in the basis of the Frenet n-frame, and use the Frenet equations]

2. (5 points) [Problem 26 from text] Let c_1 and c_2 be two plane closed curves with the property that the line segment $\overline{c_1(t)c_2(t)}$ containing them never contains the origin. Show that $W_{c_1} = W_{c_2}$. [Hint: I found that I had to use proof by contradiction to do this problem.]

3. (3 points) [Problem 27 from the text] Show the equivalence $(1) - (4)$ of 2.31 does not hold for curves which are not simply closed. [i.e. give an example of a closed curve which is not simple, where one, but not all, of $(1) - (4)$ hold. A well drawn picture will suffice.]

4. (4 points) Suppose that

$$c : [a, b] \to S^1 \subset \mathbb{R}^2$$

is a closed curve. Show that the winding number can be computed by the line integral

$$W_c = \frac{1}{2\pi} \int_c x dy - y dx.$$

Date: Assigned: 9/29/09, Due: 10/6/09.