

18.950: PSET 4

1. (3 points) [Problem 1 from the text, $n = 2$ case] Verify that the matrix $[g_{i,j}]$ of the first fundamental form of $f : U \rightarrow \mathbb{R}^3$ can be written as a matrix product $(Df)^T \circ (Df)$.
2. (5 points) Do problem 3 of section 3.
3. Suppose that $q = q(x, y)$ is a real valued function of 2-variables, and let $M \subset \mathbb{R}^3$ be the surface given by the equation

$$z = q(x, y).$$

Consider the parameterization

$$f(x, y) = (x, y, q(x, y)).$$

- (a) (2 points) Compute $[g_{i,j}]$.
- (b) (2 points) Compute the Gauss map $\nu(x, y)$.
- (c) (3 points) Compute $[h_{i,j}]$.
- (d) (3 points) Compute $[e_{i,j}]$.

Observe how these formulas simplify at a point (x_0, y_0) where $\sigma_x(x_0, y_0) = \sigma_y(x_0, y_0) = 0$. (i.e. M has a horizontal tangent plane.)