18.950: PSET 9

1. (4 points) Show that if M is an n-manifold in $\mathbb{R}^N$, and if ω is a k-form on M expressed as

$$\omega = \sum_{i_1 < \ldots < i_k} \omega_{i_1,\ldots,i_k} du^{i_1} \wedge \cdots \wedge du^{i_k}$$

in local coordinates, that $d(d\omega) = 0$.

2. (5 points) The divergence theorem states that if R is a 3-submanifold of $\mathbb{R}^3$ with boundary surface ∂M, and if

$$X(x, y, z) = (P(x, y, z), Q(x, y, z), R(x, y, z))$$

is a vector field in $\mathbb{R}^3$, that

$$\int_{\partial M} X \cdot d\vec{A} = \int_M divX\, dV.$$

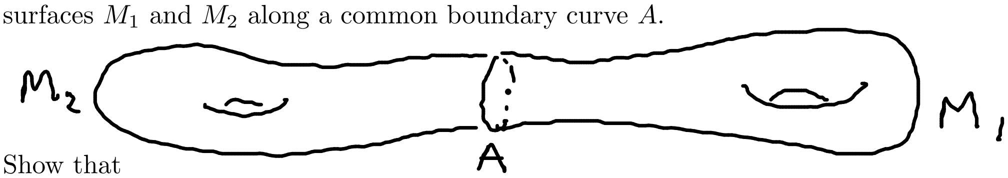
Show that this is a special instance of the generalized Stoke's theorem

$$\int_{\partial M} \omega = \int_M d\omega$$

using the form

$$\omega = Pdy \wedge dz + Qdz \wedge dx + Rdx \wedge dy.$$

3. (4 points) Suppose that M is a closed surface which is obtained by gluing two surfaces M_1 and M_2 along a common boundary curve A.



Show that

$$\chi(M) = \chi(M_1) + \chi(M_2) - \chi(A)$$

You may assume that each surface M_1 and M_2 is triangulated, and the triangulations match up (edges to edges, vertices to vertices) along the common boundary. (Here, A is 1-dimensional, the triangulations restrict to cover A with edges and vertices, and $\chi(A) := V - E$

4. (4 points) What is $\chi(D^2)$ (the 2-disk)? What is $\chi(S^1 \times [0, 1])$ (a cylinder without top or bottom)?

5. (5 points) Argue by induction on g that for the surface M given by a g-holed doughnut (a.k.a. genus g surface) the Euler characteristic is

$$\chi(M) = 2 - 2g.$$

Hint: problems 3 and 4 should be of assistance. You can get a genus $g + 1$ surface from a genus g surface by cutting out two disks, and gluing in a cylinder.

Date: Assigned: 11/24/09, Due: THURSDAY 12/3/09.