Multiple Choice

1.(7 pts.) Compute the tangent plane to the surface parametrized by $\mathbf{r} = u\mathbf{i} + uv\mathbf{j} + (u+v)\mathbf{k}$ at the point $(1, 2, 3)$.

(a) $3x + 2y + z = 10$

(b) $\langle x, y, z\rangle = \langle 1 + u, 2 + uv, 3 + u + v\rangle$

(c) $x - y + z = 2$

(d) $\dfrac{x-1}{1} = \dfrac{y-2}{2} = \dfrac{z-3}{3}$

(e) $x + 2y + 3z = 14$

2.(7 pts.) Find the directional derivative of the function $f(x, y) = \sqrt{x + 2y}$ at the point $(2, 1)$ in the direction of the vector $\mathbf{v} = \langle 1, -1\rangle$.

(a) $-\dfrac{1}{2}$ (b) $-\dfrac{\sqrt{2}}{8}$ (c) $\dfrac{1}{2}$ (d) $\dfrac{\sqrt{2}}{8}$ (e) $\langle 2, -2\rangle$

3.(7 pts.) A particle starts at the origin $(0, 0)$, moves along the x-axis to $(2, 0)$, then along the curve $y = \sqrt{4 - x^2}$ to the point $(0, 2)$, and then along the y-axis back to the origin. Use Green's Theorem to find the work done on this particle by the force field $\mathbf{F}(x, y) = y^2\mathbf{i} + 2x(y + 1)\mathbf{j}$

(a) 0 (b) $\dfrac{\pi}{2}$ (c) π (d) 2π (e) 3π

4.(7 pts.) Use the Divergence theorem to calculate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$; that is calculate the flux of $\mathbf{F}$ across S.

$$\mathbf{F} = \langle e^y, zy, xy^2\rangle,$$

S is the surface of the solid bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = 2$ and $z = 4$ with outward orientation.

(a) $\dfrac{3\pi}{2}$ (b) 6π (c) 4π (d) 2π (e) π

5.(7 pts.) Evaluate

$$\iint_R (y+x)e^{y-x}\,dA,$$

Where R is the rectangle in the xy-plane with vertices $(0,1),(1,0),(2,1),(1,2)$. (Hint: use the change of variables $u = y - x$, $v = y + x$.)

(a) $2e$ (b) $8e - \frac{8}{e}$ (c) 0 (d) $2e - \frac{2}{e}$ (e) $\frac{8}{e}$

6.(7 pts.) Which integral computes the volume of the solid bounded by $z = 4 - x^2 - y^2$ and the xy-plane in cylindrical coordinates?

(a) $\int_0^{2\pi}\int_0^2\int_0^{4-r^2} dz dr d\theta$

(b) $\int_0^{\pi}\int_0^2\int_0^{4-r^2} r\, dz dr d\theta$

(c) $\int_0^{\pi}\int_0^4\int_0^{4-r^2} dz dr d\theta$

(d) $\int_0^{\pi}\int_0^4\int_0^{4-r^2} r\, dz dr d\theta$

(e) $\int_0^{2\pi}\int_0^2\int_0^{4-r^2} r\, dz dr d\theta$

7.(7 pts.) Find the absolute maximum value of $f(x,y) = x^4 - x^2 - 4y^2$ on the solid ellipse $x^2 + 4y^2 \leq 4$

(a) 8 (b) 16 (c) 18 (d) 12 (e) 10

8.(7 pts.) Let C be the rectangle in the $z = 1$ plane with verticies $(0,0,1)$, $(1,0,1)$,$(1,3,1)$, and $(0,3,1)$ oriented counterclockwise when viewed from above. Use Stokes' Theorem to evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F} = z^2\mathbf{i} + \frac{x}{3}\mathbf{j} + xy\mathbf{k}$.

(a) 1 (b) 9/2 (c) 0 (d) 6 (e) $-3/2$

9.(7 pts.) What is the equation for the osculating plane to the curve given by $\mathbf{r}(t) = \langle t, t^2, t^3\rangle$ at the point $\langle 1,1,1\rangle$.

(a) $6x - 6y + 2z = 0$ (b) $6x - 6y + 2z = 2$

(c) $x + 2y + 3z = 5$ (d) $x + 2y + 3z = 0$

(e) $2y + 6z = 8$

10.(7 pts.) Which of the following integrals computes the distance traveled by a particle moving with position vector $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ from point $\langle 1, 1, 1 \rangle$ to $\langle 3, 9, 27 \rangle$

(a) $\int_0^3 \sqrt{1 + 4t^2 + 9t^4}\, dt$ (b) $\int_0^3 1 + 2t + 3t^2\, dt$

(c) $\int_1^3 1 + 4t^2 + 9t^4\, dt$ (d) $\int_1^3 \sqrt{1 + 2t + 3t^2}\, dt$

(e) $\int_1^3 \sqrt{1 + 4t^2 + 9t^4}\, dt$

11.(7 pts.) The solid region E is inside the sphere $x^2 + y^2 + z^2 = 2$ and above the cone $z = \sqrt{x^2 + y^2}$. Which of the following integrals evaluates $\iiint_E x\, dV$.

(a) $\int_0^{\pi/4} \int_0^{2\pi} \int_0^{\sqrt{2}} \rho^3 \sin^2 \phi \cos \theta d\rho d\theta d\phi$ (b) $\int_0^{\pi/2} \int_0^{2\pi} \int_0^{\sqrt{2}} \rho^2 \sin \phi \cos \theta d\rho d\theta d\phi$

(c) $\int_0^{\pi/4} \int_0^{2\pi} \int_0^{\sqrt{2}} \rho \sin^2 \phi \cos \theta d\rho d\theta d\phi$ (d) $\int_0^{\pi/4} \int_0^{2\pi} \int_0^{\sqrt{2}} \rho \sin \phi \cos \theta d\rho d\theta d\phi$

(e) $\int_0^{\pi/2} \int_0^{2\pi} \int_0^{\sqrt{2}} \rho^3 \sin \phi \cos \theta d\rho d\theta d\phi$

12.(7 pts.) Evaluate the intgral $\int_0^1 \int_{\sqrt{y}}^1 \cos x^3\, dxdy$ (hint: change the order of integration).

(a) $\sqrt{3}$ (b) $\frac{1}{3}$ (c) $\frac{\sin 1}{3}$ (d) $\sin 1$ (e) 1

13.(7 pts.) Suppose that the vector field $\mathbf{F} = 2xe^{yz}\mathbf{i} + zx^2e^{yz}\mathbf{j} + yx^2e^{yz}\mathbf{k}$ is conservative and C is a smooth simple curve with the starting point $(0, 0, 0)$ and end point $(1, 2, 3)$. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$.

(a) e (b) e^6 (c) 0 (d) $-e$ (e) $-e^6$

14.(7 pts.) A partical has the position function $\mathbf{r}(t)$. Suppose that at time $t = 0$, the initial position is $\langle 1, 1, 1\rangle$ and the initial velocity is $\langle 0, 0, 0\rangle$. If the partical's acceleration is $\mathbf{a}(t) = \langle 2t, e^t, 12t^2\rangle$, then find $\mathbf{r}(1)$.

(a) $\langle \frac{1}{3}, e-1, 1\rangle$ (b) $\langle \frac{4}{3}, e+1, 2\rangle$ (c) $\langle 1, e, 1\rangle$

(d) $\langle \frac{4}{3}, e-1, 2\rangle$ (e) $\langle \frac{1}{3}, e, 2\rangle$

15.(7 pts.) The surface S is the graph of the function $z = \frac{2}{3}(x^{3/2} + y^{3/2})$ for $0 \le x \le 1$, $0 \le y \le 1$. Evaluate the integral $\iint_S \frac{1}{\sqrt{1+x+y}} dS$.

(a) 2 (b) -1 (c) 0 (d) -2 (e) 1

16.(7 pts.) Which integral gives the surface area of the surface S parameterized by $\mathbf{r}(u, v) = \langle u^2 \cos v, u^2 \sin v, v\rangle$, where $0 \le u \le 1, 0 \le v \le \pi$.

(a) $\int_0^\pi \int_0^1 2u\sqrt{1+u^4}\, dudv$ (b) $\int_0^\pi \int_0^1 (4u^2 + 4u^6)\, dudv$

(c) $\int_0^\pi \int_0^1 4u^2(\sin v + \cos v) + 4u^4\, dudv$ (d) $\int_0^\pi \int_0^1 2u\sqrt{1+u^2}\, dudv$

(e) $\int_0^\pi \int_0^1 \sqrt{4u^2\sin^2 v - \cos^2 v + 4u^6}\, dudv$

17.(7 pts.) Let $f(x, y) = 3x - x^3 - 2y^2 - y^4$. According to the second derivative test, which one of the following is true?

(a) The function has 1 local maxima and 1 local minima.

(b) The function has 1 saddle point and 1 local minima.

(c) The function has 2 local maxima.

(d) The function has 1 local maxima and 1 saddle point.

(e) The function has 2 saddle points.

18.(7 pts.) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = x\mathbf{i} - z\mathbf{j} + y\mathbf{k}$ and C is given by $\mathbf{r}(t) = 2t\mathbf{i} + 3t\mathbf{j} - t^2\mathbf{k}$ with $-1 \leq t \leq 1$.

(a) -2 (b) 0 (c) 6 (d) 2 (e) -6

19.(7 pts.) Which of the following is the tangent plane to the graph $z = e^{-xy} \sin x$ at the point $(\pi, 0, 0)$.

(a) $z = -x + \pi y + \pi$

(b) $z = -x - y + \pi$

(c) $z = 0$

(d) $z = -x - \pi y + \pi$

(e) $z = -x + \pi$

20.(7 pts.) Find the flux of the vector field

$$\mathbf{F}(x, y, z) = y\mathbf{i} - x\mathbf{j} + z\mathbf{k}$$

over a surface with **downward** orientation, whose parametric equation is given by

$$\mathbf{r}(u, v) = 2u\mathbf{i} + 2v\mathbf{j} + (5 - u^2 - v^2)\mathbf{k}$$

with $u^2 + v^2 \leq 1$.

(a) $-\frac{56\pi}{3}$ (b) $\frac{112\pi}{3}$ (c) -18π (d) -36π (e) 9π