Multiple Choice

1.(7 pts.) Which integral gives the mass of the solid bounded by $z = x^2 + y^2$ and $z = 4$ with density function $\rho(x, y, z) = z^2$.

(a) $\int_0^{2\pi} \int_0^2 \int_4^{r^2} z^2 r \, dz dr d\theta$

(b) $\int_0^{2\pi} \int_0^2 \int_{r^2}^4 z^2 r \, dz dr d\theta$

(c) $\int_0^{2\pi} \int_0^2 \int_{x^2+y^2}^4 z^2 r \, dz dr d\theta$

(d) $\int_0^{2\pi} \int_0^2 \int_{x^2+y^2}^4 z^2 \, dz dr d\theta$

(e) $\int_0^{2\pi} \int_0^2 \int_{r^2}^4 z^2 \, dz dr d\theta$

2.(7 pts.) Which integral gives the surface area of the surface S parametrized by $\mathbf{r}(u, v) = \langle u\cos(v), u\sin(v), u^5\rangle$ where $0 \le u \le 1$ and $0 \le v \le \pi/2$.

(a) $\int_0^{\pi/2} \int_0^1 u^2 \sqrt{25u^8 + 1} \, du dv$

(b) $\int_0^{\pi/2} \int_0^1 \left(-5u^5(\cos(v) + \sin(v)) + u\right) du dv$

(c) $\int_0^{\pi/2} \int_0^1 u\sqrt{25u^8 + 1} \, du dv$

(d) $\int_0^{\pi/2} \int_0^1 \, du dv$

(e) $\int_0^{\pi/2} \int_0^1 \left(25u^{10} + u^2\right) du dv$

3.(7 pts.) Compute the tangent plane to the surface parametrized by $\mathbf{r}(u, v) = \langle 4 - u^2 - v^2, 2u, v\rangle$ at the point $(2, 2, 1)$.

(a) $2x + 2y + z = 9$

(b) $\dfrac{x-2}{2} = \dfrac{y-2}{2} = \dfrac{z-1}{4}$

(c) $2x + 2y + 4z = 12$

(d) $\langle x, y, z\rangle = \langle 2t - 4, 2t - 4, 4t - 4\rangle$

(e) $x + y + 2z = 0$

4.(7 pts.) Which of the integrals computes $\iiint_E z \, dV$ where E is the region bounded by the spheres $x^2 + y^2 + z^2 = 4$, $x^2 + y^2 + z^2 = 9$ and above the cone $z = \sqrt{x^2 + y^2}$.

(a) $\int_0^{2\pi}\int_0^{\pi/4}\int_2^3 \rho^3\sin(\varphi)\cos(\varphi)\,d\rho d\varphi d\theta$ (b) $\int_0^{2\pi}\int_0^{\pi/2}\int_2^3 \rho\cos(\varphi)\,d\rho d\varphi d\theta$

(c) $\int_0^{2\pi}\int_0^{\pi/2}\int_2^3 \rho^3\sin(\varphi)\cos(\varphi)\,d\rho d\varphi d\theta$ (d) $\int_0^{2\pi}\int_0^{\pi/4}\int_2^3 \rho\cos(\varphi)\,d\rho d\varphi d\theta$

(e) $\int_0^{2\pi}\int_0^{\pi/4}\int_2^3 \rho\sin(\varphi)\,d\rho d\varphi d\theta$

5.(7 pts.) Find the absolute maximum value of $f(x,y) = x^2 + x + 2y^2$ on the disc $x^2 + y^2 \leq 1$.

(a) 2 (b) $\frac{9}{4}$ (c) $\frac{3+\sqrt{2}}{2}$ (d) $\frac{11}{4}$ (e) $-\frac{1}{4}$

6.(7 pts.) Let $f(x,y) = x^3 + y^3 - 3x - 3y - 3$. Which one of the following is true?

(a) The function has one saddle points. (b) The function has no saddle points.

(c) The function has two local minima. (d) The function has two saddle points.

(e) The function has two local maxima.

7.(7 pts.) Which is the largest value that the direction derivatives of $f(x,y,z) = x^2 + 2xy + z^2$ can have at the point $(1,0,1)$?

(a) $\sqrt{3}$ (b) 2 (c) $\sqrt{12}$ (d) $\sqrt{8}$ (e) 4

8.(7 pts.) Evaluate the integral

$$\int_0^1 \int_{e^x}^{e} \frac{1}{\ln y} dy dx$$

(Hint: Reverse the order of integration)

(a) e (b) 1 (c) -1 (d) $e-1$ (e) 0

9.(7 pts.) Let C be a simple closed curve that lies in the xy–plane. Assume that the area of the region enclosed by the curve is 2. The curve C is oriented counterclockwise when viewed from above. Use Stokes' Theorem to evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r},$$

where $\mathbf{F} = \frac{z}{2}\mathbf{i} + \frac{x}{3}\mathbf{j} + \frac{y}{4}\mathbf{k}$

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) 1 (e) 2

10.(7 pts.) Evaluate the line integral

$$\int_C x^2dx + y^2dy + z^2dz$$

where C is parametrized by $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ for $0 \le t \le 1$.

(a) 2 (b) 0 (c) -1 (d) -2 (e) 1

11.(7 pts.) Use Green's theorem to evaluate

$$\int_C (2y + \cos x^2)dx + \sin y^2 dy,$$

where C is the triangle with vertices $(1, 0), (0, 2), (-1, 0)$ oriented counterclockwise.

(a) -4 (b) 1 (c) 3 (d) -2 (e) 0

12.(7 pts.) Use the Fundamental Theorem of Line Integrals to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = e^{x+y^2}\mathbf{i} + (1 + 2ye^{x+y^2})\mathbf{j}$ and C is the curve given by $\mathbf{r}(t) = \langle t^{1/2}, t \rangle$ for $0 \le t \le 1$.

(a) 0 (b) $-e^2$ (c) $-e$ (d) e^2 (e) e

13.(7 pts.) Compute the normal component of the acceleration at $t = 0$ for the parametric curve $\mathbf{r}(t) = \langle e^t, \cos(t), t^2 \rangle$.

(a) 2 (b) $\sqrt{3}$ (c) $\sqrt{2}$ (d) 1 (e) $\sqrt{5}$

14.(7 pts.)Find the tangent line to the intersection curve of the surfaces $x^3 + yz = 1$ and $x^2 y = z$ at the point $(-1, 2, 1)$.

(a) $\langle t-1, -5t+2, 7t+1\rangle$ (b) $\langle t-1, 5t+2, -3t+1\rangle$

(c) $\langle -3t-1, -5t+2, 7t+1\rangle$ (d) $\langle -3t+1, -5t-2, 7t-1\rangle$

(e) $\langle 7t-1, -3t+2, -5t+1\rangle$

15.(7 pts.) Calculate the surface integral $\iint_S \mathbf{F}\, d\mathbf{S}$; that is, calculate the flux of $\mathbf{F}$ across S.

$$\mathbf{F} = \langle 2ye^z, z^2 - y, 3\cos(x)\rangle,$$

S is the surface of the solid bounded by the planes $y + z = 2$, $x = 3$ and coordinate planes $x = 0$, $y = 0$, $z = 0$. The surface S is endowed with the **inward** orientation.

(a) $2 - e$ (b) -3 (c) 6 (d) 4 (e) $-e$

16.(7 pts.) Find $\mathbf{r}(1)$, the position of a particle at time $t = 1$, if the acceleration is $\mathbf{a}(t) = \langle 2t+1, 3t^2, 0\rangle$, knowing that $\mathbf{v}(0) = \langle 1, 0, 2\rangle$ and $\mathbf{r}(0) = \langle 0, -1, -2\rangle$.

(a) $\langle \frac{13}{6}, -1, 0\rangle$ (b) $\langle -\frac{1}{6}, -\frac{1}{2}, -2\rangle$ (c) $\langle \frac{13}{6}, 1, -2\rangle$

(d) $\langle \frac{11}{6}, -\frac{3}{4}, 0\rangle$ (e) $\langle \frac{11}{6}, -\frac{1}{4}, 0\rangle$

17.(7 pts.) The position vector of a particle is given by $\mathbf{r}(t) = \langle \cos(\pi t/2), \sin(\pi t/2), t^2\rangle$. Which of these integrals calculates the distance travelled by the particle in going from $\langle 1, 0, 0\rangle$ to $\langle 0, 1, 1\rangle$?

(a) $\int_0^1 \sqrt{\frac{\pi}{2}(\cos(\pi t/2) + \sin(\pi t/2))}\, dt$ (b) $\int_0^1 \sqrt{\frac{\pi^2}{4} + 4t^2}\, dt$

(c) $\int_0^1 \sqrt{1 + 4t^2}\, dt$ (d) $\int_0^1 \sqrt{\frac{1}{4} + 4t^2}\, dt$

(e) $\int_{-\pi}^{\pi} (1 + 4\tan^2 t)\, dt$

18.(7 pts.) Find the flux of the electric field

$$\mathbf{E}(x,y,z) = \frac{x}{x^2+y^2}\mathbf{i} + \frac{y}{x^2+y^2}\mathbf{j} + \mathbf{k}$$

over a surface with **downward** orientation, whose parametric equation is given by

$$\mathbf{r}(u,v) = u\mathbf{i} + v\mathbf{j} + (1-u^2-v^2)\mathbf{k}, \text{ with } u^2+v^2 \leq 1.$$

(a) $-\pi$ (b) 4π (c) -3π (d) 0 (e) 2π

19.(7 pts.) Evaluate

$$\iint_R \frac{1}{\sqrt{xy}}\, dA,$$

where R is the region in the xy-plane bounded by $\sqrt{x}+\sqrt{y} \leq 1$, $x \geq 0$ and $y \geq 0$.
(Hint: use the change of variables $x = u^2$ and $y = v^2$)

(a) 2 (b) $\frac{3}{2}$ (c) 4 (d) 1 (e) 16

20.(7 pts.) Let S be the part of the surface $z = xy$, that lies above the square $0 \leq x \leq 1$, $0 \leq y \leq 1$. Which one of the following integrals equals

$$\iint_S z\, dS.$$

(a) $\int_0^1 \int_0^1 xy\sqrt{1+4x^2+4y^2}\, dx\, dy$

(b) $\int_0^1 \int_0^1 xy(x^2+y^2)\, dx\, dy$

(c) $\int_0^1 \int_0^1 xy(x+y)\, dx\, dy$

(d) $\sqrt{3}\int_0^1 \int_0^1 xy\, dx\, dy$

(e) $\int_0^1 \int_0^1 xy\sqrt{1+x^2+y^2}\, dx\, dy$