Name: ____________________________

Instructor: ________________________

Math 20550, Last Year Exam 3
April 23, 2019

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 minutes
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 9 pages of the test.
- Each multiple choice question is 6 points. Each partial credit problem is 12 points. You will receive 4 extra points.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	________
11.	________
12.	________
13.	________
Extra Points.	4
Total:	________

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Multiple Choice

1.(6 pts)Use cylindrical coordinates to evaluate $\iiint_E (x^2 + y^2)\, dV$, where

$$E = \{(x, y, z) \mid \sqrt{x^2 + y^2} \leq z \leq 2\}.$$

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$ (c) $\frac{4\pi}{9}$ (d) $\frac{16\pi}{5}$ (e) $\frac{4\pi}{3}$

2.(6 pts) Evaluate $\int_C xy\, ds$, where C is given by $\vec{r}(t) = \langle 4\cos t, 4\sin t, 3t\rangle$ for $0 \leq t \leq \frac{\pi}{2}$.

(a) 10 (b) 40 (c) 5 (d) 0 (e) -40

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3.(6 pts) Find the total mass of the laminated (i.e., thin) region D having density $\rho(x, y) = \sqrt{x^2 + y^2}$, where

$$D = \{(x, y) \mid x^2 + y^2 \leq 4, y \geq 0\}.$$

(a) $\frac{4\pi}{3}$ (b) $\frac{8\pi}{3}$ (c) $\frac{3\pi}{2}$ (d) $\frac{4}{3}$ (e) $\frac{2\pi}{3}$

4.(6 pts) Use spherical coordinates to evaluate $\iiint_E e^{(x^2+y^2+z^2)^{3/2}} dV$, where

$$E = \{(x, y, z) \mid y \geq 0, z \geq 0, x^2 + y^2 + z^2 \leq 1\}.$$

(a) $4\pi e$ (b) 0 (c) $\frac{\pi}{3}e$ (d) $\frac{\pi}{3}(e-1)$ (e) $\frac{4\pi}{3}(e-1)$

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5.(6 pts) Let $\vec{F} = \langle xz, xyz, -y^2 \rangle$. Compute $\operatorname{curl} \vec{F}$.

(a) $\langle -y(2+x), x, yz \rangle$ (b) 0 (c) $z + xy$

(d) $\langle x, -y(2+x), yz \rangle$ (e) $\langle -y(2+x), -x, yz \rangle$

6.(6 pts) Use a double integral to find the area enclosed by one loop of the four-leaved rose $r = \sin(2\theta)$. The region inside the loop is described in polar coordinates by $0 \le \theta \le \dfrac{\pi}{2}$ and $0 \le r \le \sin(2\theta)$.

(a) $\dfrac{\pi}{2}$ (b) 0 (c) $-\dfrac{\pi}{2}$ (d) $\dfrac{\pi}{8}$ (e) $\dfrac{\pi}{4}$

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7.(6 pts) Find the work $\int_C \vec{F} \cdot d\vec{r}$ done by the force field $\vec{F} = \langle xy, yz, zx \rangle$ in moving a particle along the curve C given by $\vec{r}(t) = \langle t, t^2, t^3 \rangle$ for $-1 \leq t \leq 1$.

(a) $\frac{1}{4}$ (b) $\frac{5}{7}$ (c) $\frac{1}{2}$ (d) $\frac{10}{7}$ (e) $\frac{27}{28}$

8.(6 pts)Use Green's theorem to evaluate $\int_C \Big((3y - e^{x^2})dx + (7x + \sqrt{y^{99} + y + 100})dy \Big)$ where C is the circle $x^2 + y^2 = 9$ with the counter-clockwise orientation.

(a) 3π (b) 36π (c) 0 (d) -36π (e) 7π

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9.(6 pts) Let $x = 2u$ and $y = -3v$. Then $\int_{-3}^{3}\int_{-2}^{2} f(x, y)dxdy$ can be written as:

(a) $\frac{1}{6}\int_{-1}^{1}\int_{-1}^{1} f(2u, -3v)dudv$

(b) $6\int_{-3}^{3}\int_{-2}^{2} f(2u, -3v)dudv$

(c) $6\int_{-1}^{1}\int_{-1}^{1} f(2u, -3v)dudv$

(d) $-4\int_{-1}^{1}\int_{-1}^{1} f(2u, -3v)dudv$

(e) $-6\int_{-1}^{1}\int_{-1}^{1} f(2u, -3v)dudv$

10.(6 pts)Which of the following vector fields cannot be written as curl $\vec{F}$?

(a) $\langle -x - y + 1, xy - 1, -xz + y + z\rangle$

(b) $\langle -y, -z, -x\rangle$

(c) $\langle -y\cos(z), -z\cos(x), -x\cos(y)\rangle$

(d) $\langle 2yz, xyz, 3xy\rangle$

(e) $\langle 1 - 2z, 1 - 2x, 1 - 2y\rangle$

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Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(12 pts.)Let $\vec{F} = \langle y^2 + 1, 2xy + 2y + e^{3z}, 3ye^{3z} + 3z^2 \rangle$.

(a) Find $\operatorname{curl} \vec{F}$.

(b) Find f such that $\nabla f = \vec{F}$.

(c) Evaluate $\displaystyle\int_C \vec{F} \cdot d\vec{r}$ where C is any smooth curve beginning at $(1, 0, 0)$ and ending at $(0, 1, 0)$.

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12.(12 pts.)Let E be the tetrahedron enclosed by the coordinate planes $x = 0$, $y = 0$, $z = 0$ and the plane $2x + y + z = 2$. Assume the density function is $\rho(x, y, z) = 1$. Write an iterated integral (with limits) for the moment of the solid E about the yz-plane. (You do NOT need to compute this iterated integral.)

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13.(12 pts.)Use the transformation $x = \sqrt{3}u - v$, $y = \sqrt{3}u + v$ to evaluate the integral $\iint_R (x^2 - xy + y^2)dA$, where R is the region bounded by the ellipse $x^2 - xy + y^2 = 3$.