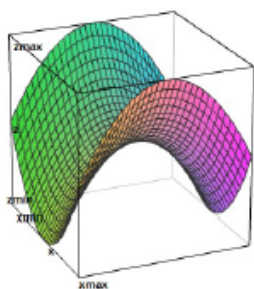**M20550 Calculus III Tutorial**
Worksheet 4

1. Find and sketch the domain of the function
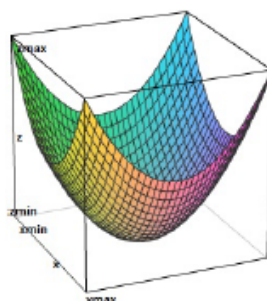
$$f(x, y) = \frac{\ln\left(x^2 + 4y^2 - 4\right)}{9 - x^2}.$$

2. Select the correct graph and the correct contour plot of level curves for the function

$$f(x, y) = x^2 - y^2$$
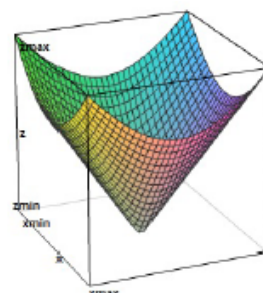
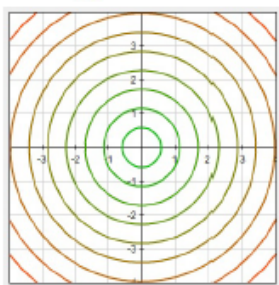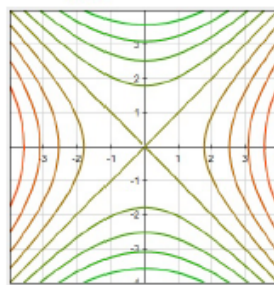I.   II.   III.
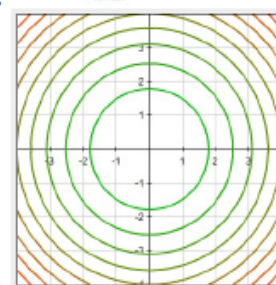
A.   B.   C.

(a) I and B   (b) I and A   (c) II and A

(d) II and C   (e) III and C

3. Evaluate the following limit

$$\lim_{(x,y)\to(0,0)} \frac{y + xe^{-y^2}}{1 + x^2}.$$

4. Show that the following limit does not exist

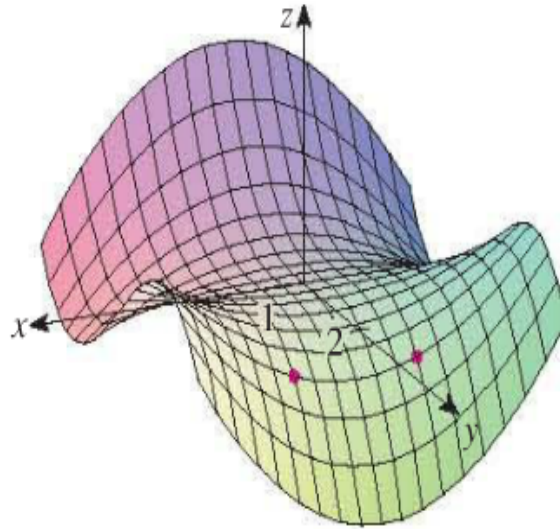$$\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4 + y^2}.$$

(HINT: To prove that the limit does not exist, find two different curves in the plane passing through $(0,0)$ and take the limit along each of them. Example: $r(t) = (t, 0)$. Try another one!)

5. Find the second partial derivative g_{xy} of the function

$$g(x, y) = x^3 y^2 + e^{xy}.$$

6. The graph of f is shown below

   Determine the sign of

   (a) $f_x(1, 2)$

   (b) $f_y(1, 2)$

   (c) $f_x(-1, 2)$

   (d) $f_y(-1, 2)$

7. The paraboloid $z = 6 - x - x^2 - 2y^2$ intersects the plane $x = 1$ in a parabola. Use the geometry of partial derivative to find the **slope** for the tangent line to this parabola at the point $(1, 2, -4)$.