Name: Date: 03/05/2020

M20550 Calculus III Tutorial
Worksheet 6

1. *(The D-formula)* Find the local maximum and the local minimum value(s) and saddle point(s) of the function $z = x^3 + y^3 - 3xy + 1$.

2. Evaluate the double integral $\iint_R (4 - 2y)dA$, for $R = [0, 1] \times [0, 1]$, by identifying it as the volume of a solid.

3. Evaluate the iterated integral.

   (a) $\int_0^2 \int_0^\pi r \sin^2 \theta \; d\theta dr$

   (b) $\iint_R ye^{-xy} dA$ on $R = [0, 2] \times [0, 3]$

4. Find the volume of the solid in the first octant bounded by the cylinder $z = 16 - x^2$ and the plane $y = 5$.

5. *(Double integrals over general regions)* Evaluate the following integrals:

   (a) $\iint_D xydA$, D is enclosed by the curves $y = x^2$, $y = 3x$;

   (b) $\iint_D ydA$, D is bounded by $y = x - 2$, $x = y^2$.

6. *(Fubini's theorem)* Change the order of integration in the following integrals:

   (a) $\int_0^2 dx \int_x^{2x} f(x, y)dy$;

   (b) $\int_{-6}^2 dx \int_{\frac{x^2}{4}-1}^{2-x} f(x, y)dy$;

   Hint: in the second case you may need to sketch the region and to split the integral into two integrals over smaller regions.

7. *(Optional: Lagrange multipliers with two constraints)* Find the maximum value of the function $f(x, y, z) = x + 2y$ on the curve of intersection of the plane $x + y + z = 1$ and the cylinder $y^2 + z^2 = 4$.