Name: Date: 26/03/2020

M20550 Calculus III Tutorial
Worksheet 7

1. Evaluate the given integral.
$$\iint_R \arctan\left(\frac{y}{x}\right)\, dA$$
where $R = \{(x, y) : 1 \leq x^2 + y^2 \leq 4, 0 \leq y \leq x\}$.

2. (a) Let E_1 be the solid that lies under the plane $z = 1$ and above the region in the xy-plane bounded by $x = 0$, $y = 0$, and $2x + y = 2$. Write the triple integral $\iiint_{E_1} xz\, dV$ but do not evaluate it.

   (b) Let E_2 be the solid region in the first octant that lies under the paraboloid $z = 2 - x^2 - y^2$. Write the triple integral $\iiint_{E_2} xz\, dV$ in cylindrical coordinates (you don't need to evaluate it).

3. Find the center of mass of the solid S bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 1$ if S has constant density 1 and total mass $\frac{\pi}{2}$. (Hint: $\overline{x}$ and $\overline{y}$ can be found by symmetry of the solid being considered).

4. Find the volume of the solid enclosed by the paraboloid $z = x^2 + y^2$ and the plane $z = 1$.

5. Use polar coordinates to show that
$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{-(x^2+y^2)} dA = \pi$$
and deduce that $\int_{-\infty}^{+\infty} e^{-x^2} dx = \sqrt{\pi}$.

6. The plane $x + y + 2z = 2$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse. Find the points on the ellipse that are nearest and farthest from the origin.

7. Set up, but do not solve, the integral that gives the volume of the solid region bounded by the paraboloid $z = 3x^2 + 3y^2$ and the cone $z = 4 - \sqrt{x^2 + y^2}$.