Name: Date: 04/02/2019

M20550 Calculus III Tutorial
Worksheet 7

1. Using spherical coordinates, compute the volume, $V(R)$ of a sphere of radius R.

2. Now compute the surface area, $A(R)$, of a sphere of radius R. Hint: Recall the Fundamental Theorem of Calculus:

$$\frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x).$$

   And recall the common problem from single variable calculus where you have to find the volume of a water tank of height h by integrating the cross sectional area, $A(y)$, over the height.

$$Volume(Tank) = \int_0^h A(y)dy$$

   We have a similar formula for the volume of the sphere;

$$V(R) = \int_0^R A(\rho)d\rho.$$

3. Let E_3 be the solid region that lies above the cone $z = \sqrt{x^2+y^2}$ and below the plane $z = 2$. Write the triple integral $\iiint_{E_3} xz\,dV$ in spherical coordinates (you don't need to evaluate it).

4. Find the mass of the solid between the spheres $x^2+y^2+z^2 = 1$ and $x^2+y^2+z^2 = 4$ whose density is $\delta(x, y, z) = x^2 + y^2 + z^2$.

5. In this problem, we are going to calculate the same integral in two different ways by changing coordinates. Compute the following integral;

$$\int_0^1 \int_0^1 x^3 y dx dy$$

   first, by making the coordinate change $u = x^2, v = xy$, and then as you normally would. (Don't forget to multiply by the Jacobian!)

6. Let R be the parallelogram enclosed by the lines $x + 3y = 0$, $x + 3y = 2$, $x + y = 1$, and $x + y = 4$. Evaluate the following integral by making appropriate change of variables

$$\iint_R \frac{x+3y}{(x+y)^2}\,dA.$$

7. Evaluate the line integral $\int_C (z-2xy)\, ds$ along the curve C given by $\mathbf{r}(t) = \langle \sin t, \cos t, t \rangle$, $0 \le t \le \frac{\pi}{2}$.

8. Find $\int_C 2xy^3\, ds$ where C is the upper half of the circle $x^2 + y^2 = 4$.