Name: Date: 04/08/2020

M20550 Calculus III Tutorial
Worksheet 9

1. Calculate the line integral $\displaystyle\int_C (y^2+x)\,dx+4xy\,dy$ where C is the arc of $x=y^2$ from $(1,1)$ to $(4,2)$.

2. Evaluate the line integral $\displaystyle\int_C z^2\,dx+x\,dy+y\,dz$ where C is the line segment from $(1,0,0)$ to $(4,1,2)$.

3. Compute $\displaystyle\int_C x^2\,ds$ where C is the intersection of the surface $x^2+y^2+z^2=4$ and the plane $z=\sqrt{3}$.

4. Determine whether or not the following vector fields are conservative:
   (a) $\mathbf{F}=(3+2xy)\,\mathbf{i}+(x^2-3y^2)\,\mathbf{j}$
   (b) $\mathbf{F}=\mathbf{i}+\sin z\,\mathbf{j}+y\cos z\,\mathbf{k}$

5. Evaluate $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y,z)=-2xy\,\mathbf{i}+4y\,\mathbf{j}+\mathbf{k}$ and $\mathbf{r}(t)=t\,\mathbf{i}+t^2\,\mathbf{j}+\mathbf{k}$, $0\le t\le 2$.

6. Evaluate $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}=(y^2\cos(xy^2)+3x^2)\,\mathbf{i}+(2xy\cos(xy^2)+2y)\,\mathbf{j}$ is a conservative vector field and C is any curve from the point $(-1,0)$ to $(1,0)$.

7. Use Green's Theorem to evaluate

$$\int_C \left(-\frac{y^3}{3}+\sin x\right)\,dx+\left(\frac{x^3}{3}+y\right)\,dy,$$

   where C is the circle of radius 1 centered at $(0,0)$ oriented counterclockwise when viewed from above.