HW1: BONUS PROBLEM

Recall that we proved Euler's theorem:

Theorem 1. *Let S^2 be a sphere with a polygonal decomposition, with V vertices, E edges, and F faces. Then the Euler characteristic*

$$\chi(S^2) = V - E + F$$

is equal to 2.

A platonic solid is a polyhedron where every side is a q-gon, and every vertex intersects the same number of edges (say every vertex intersects p edges). On the first day of class I pointed out that every platonic solid gives rise to a polygonal decomposition of S^2 (by "inflating" it to make it a ball). Prove that the only possible Platonic solids are the tetrahedron, cube, octahedron, dodecahedron, and icosahedron, using Euler's theorem. (Hint: first show that $2E = qF$ and $2E = pV$ — this plus Euler's equation, and the fact that p, q and F must all be integers greater than or equal to 3, puts severe limitations on the possible values of p, q, and F.)

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