5-Thom spectra, ring spectra

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Thom Spectra X = finite CW cx,

$\xi \in KO^0(X)$ $\xi = [V]-[\mathbb{R}^N]$

Define $X^\xi =$ " $\Sigma^{-N}\Sigma^\infty X^V$ " $= (X^V, N)$

For general X $X = \lim X^{[k]}$ $\xi|_{X^{[k]}} = [V_k] - \mathbb{R}^{N_k}$ $X^\xi := \varinjlim (X^{V_k}, N_k)$

Ring Spectra: $E \wedge E \xrightarrow[\mu]{} E$

$\eta: S \to E$

$$\begin{tikzcd} S\wedge E \arrow[r] \arrow[dr, equal] & E\wedge E \arrow[d, "\mu"] & E\wedge S \arrow[l] \arrow[dl, equal] \\ & E & \end{tikzcd}$$

I, SHC

assoc.

comm.

Note! $\pi_* X \otimes \pi_* Y \longrightarrow \pi_*(X\wedge Y)$

[This is Not an iso]

E = Ring spectrum $\Longrightarrow$ E_* is a ring.

& $E^*(X)$ is a ring & X

$$\left(E^{X_+} \wedge E^{X_+} \longrightarrow (E\wedge E)^{X\times X_+} \xrightarrow{\Delta^*} (E\wedge E)^{X_+} \xrightarrow[\mu]{} E^{X_+} \right)$$

So $\pi_* E^{X_+}$ is a ring.

$E_i \wedge E_j \longrightarrow E_{i+j}$ (e.g. Sphere spectrum $\Sigma^\infty X_+$ X = H-space)

e.g. R = ring $K(R,n)\wedge K(R,m) \longrightarrow K(R,n+m)$

bottom htpy gp = bottom homology gp
= $R \otimes R$

$$\begin{array}{c} \mathrm{Hom}(R \otimes R, R) \\ \uparrow \\ H^{n+n}(K(n) \wedge K(n)) \\ \uparrow \\ \mathrm{Ext}^1(H_{n+n-1}, R) \end{array}$$

K-theory
Bordism } → ring spectra.

$(\mathcal{C}, \oplus, \otimes)$ two different symmetric monoidal structures

$K(\mathcal{C})$ is a ring.

Hurewicz Homomorphism

E ring spectrum:

$$\pi_* X \longrightarrow E_* X$$