

5-Thom spectra, ring spectra

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Thom Spectrum

$$X = \text{fibre of } \alpha$$

$$\xi \in KO^0(X)$$

$$\xi = [V] - [R^m]$$

Defn $X^\xi = \sum^{-N} \sum^m X^V = (X^V, N)$

For general X

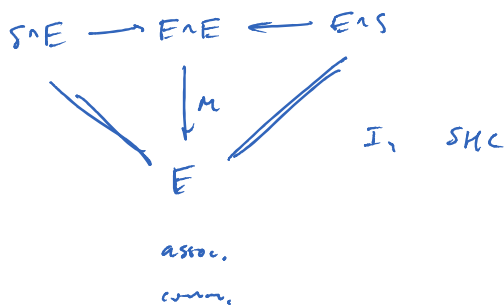
$$X = \lim X^{[k]} \quad \xi|_{X^{[k]} = [V_k] - R^{m_k}}$$

$$X^\xi := \varinjlim (X^{V_k}, N_k)$$

Ring Spectra!

$$E \wedge E \xrightarrow{\mu} E$$

$$\eta: S \rightarrow E$$



Note!

$$\pi_* X \otimes \pi_* Y \rightarrow \pi_*(X \wedge Y)$$

[This is Not an iso]

$$E = \text{Ring spectrum} \implies E_* \text{ is a ring.}$$

$$\dagger E^*(X) \text{ is a ring } \dagger X$$

$$\left(E^{X_+} \wedge E^{X_+} \xrightarrow{\mu} (E \wedge E)^{X_+ \times X_+} \xrightarrow{\delta} (E \wedge E)^{X_+} \xrightarrow{\mu} E^{X_+} \right)$$

$$\text{So } \pi_* E^{X_+} \text{ is a ring.}$$

$$E_i \wedge E_j \rightarrow E_{i+j} \quad \left(\text{e.g. Sphere spectra } \sum^{\infty} X_+ \quad X = H\text{-spaces} \right)$$

e.g.

$$R = \text{ring}$$

$$K(R, n) \wedge K(R, m) \rightarrow K(R, n+m)$$

\uparrow
 better litig sp = better homing sp
 = $R \otimes R$

$$\begin{array}{c}
 \text{Hom}(R \otimes R, R) \\
 \uparrow \\
 H^{**}(K(n) \wedge K(n)) \\
 \uparrow \\
 \text{Ext}^*(\text{Hurewicz}, R)
 \end{array}$$

K -thy }
 Bordism } n/y spectra.

$(\mathcal{C}, \oplus, \otimes)$ two different symmetric monoidal structures

$K(\mathcal{C})$ is a ring.

Hurewicz Homomorphism

E n/y spectra:

$$\pi_* X \rightarrow E_* X$$