Intermediate Topology/Geom pset 1

Assigned: 9/2/16

"Due": 9/9/16

Basic problems (required)

1) The complex projective space CP^2 admits a CW complex structure:
$$CP^2 = S^2 \cup_\eta e^4$$
Use the cohomology of CP^2 to deduce that the attaching map
$$\eta: S^3 \to S^2$$
is not null homotopic. [Hint: if it were, there would be a homotopy equivalence $CP^2 \simeq S^2 \vee S^4$]
Thus $\pi_3(S^2) \neq 0$.

2) Use this to deduce that relative homotopy groups do not satisfy excision.

3) Show that any simply connected 3-manifold is homotopy equivalent to S^3.

Less basic problems (optional)

4) Give an example of a weak homotopy equivalence which is not a homotopy equivalence.

5) Give an example of a homology isomorphism which is not a weak homotopy equivalence.