PSET 2

Problems marked (basic) are required, problems marked (less basic) are optional. You may assume the less basic problems to do the basic problems.

For X a based space with basepoint $*$, the *path space* is the space

$$PX = \{\gamma : I \to X \ : \ \gamma(0) = *\}$$

(given the subspace topology of the mapping space). The (based) loop space ΩX is the space

$$\Omega X = \{\gamma \in PX \ : \ \gamma(1) = *\}.$$

For the problems below I encourage you to not think too much about the point set topology unless you want to.

1) (less basic) The evaluation map

$$\begin{aligned} ev : PX &\to X \\ \gamma &\to \gamma(1) \end{aligned}$$

is a fibration with fiber ΩX.

2) (basic) Show that PX is contractible, and that there is an isomorphism

$$\pi_i(\Omega X) \cong \pi_{i+1}(X).$$

3) (basic) Argue that any fibration $f : E \to X$ has a map g making following diagram commute "weak parallel transport":

$$\begin{array}{ccc} PX & \xrightarrow{ev} & X \\ {\scriptstyle g}\downarrow & & \Vert \\ E & \xrightarrow[f]{} & X \end{array}$$

[Hint, lift the homotopy $H : PX \times I \to X$ given by $H(\gamma, t) = \gamma(t)$] Conclude that if E is weakly homotopy equivalent to a point, then the fiber F of f is weakly homotopy equivalent to ΩX. In particular, $\Omega BG \simeq G$.

4) (less basic) If H is a sub-Lie group of a Lie group G, then $G \to G/H$ is a locally trivial bundle with fiber H. (Assume G is compact if this makes it easier.)

5) (basic) If $EG \to BG$ is a universal G-bundle, and G is a Lie group with H a sub-Lie group H then $EG/H \simeq BH$, giving a locally trivial bundle $BH \to BG$ with fiber G/H.

Date: Assigned 9/10/16, "due" 9/16/16.