Intermediate Topology/Geom pset 5

Assigned: 9/30/16

"Due": 10/7/16

Basic problems (required)

1) Do problem 4-C at the end of Section 4 of Milnor and Stasheff (available in the google drive).

2) Show that the simplicial set Δ^2 is not a Kan complex.

Less basic problems (optional)

3) Given a simplicial set X, let $(C_*(X), \partial)$ denote the chain complex with

$$C_n(X) = \mathbb{Z}X_n \text{ (The free abelian group generated by } X_n\text{)}$$

$$\partial(x) = \sum_i (-1)^i d_i(x)$$

Verify that $\partial^2 = 0$. It turns out that

$$H_*\big(C_*(X)\big) = H_*(|X|)$$

(though this requires a sneaky trick or two to prove)

4) For G a discrete group, X a right G-set, and Y a left G-set, define the two sided bar construction to be the simplcial set $B(X, G, Y)$ with

$$B(X, G, Y)_n = X \times G^n \times Y$$

With simplicial structure maps

$$d_i(x, g_1, \dots, g_n, y) = \begin{cases} (xg_1, g_2, \dots, g_n, y), i = 0 \\ (x, g_1, \dots, g_{n-1}, g_n y), i = n \\ (x, g_1, \dots, g_i g_{i+1}, \dots, g_n, y), otherwise \end{cases}$$

$$s_i(x, g_1, \dots, g_n, y) = (x, g_1, \dots g_i, 1, g_{i+1}, \dots, g_n, y)$$

Show that $|B(G, G, *)|$ is a free contractible G-CW complex, and that the quotient is $|B(*, G, *)|$. Conclude that the latter is a model for BG. In fact, $C_*(B(*, G, *)$ is the "Bar complex" for computing group homology. This is one way to see that $H_*(BG) = H_*(G; \mathbb{Z})$ (group homology).