PSET 7

ASSIGNED 10/31/16, "DUE": 11/4/16

1. In Xiaoxiao's first talk, he asserted:

Proposition 0.1. If $\mathcal{F}$ and $\mathcal{E}$ are two stable holomorphic vector bundles over a Riemann surface (aka holomorphic curve) Σ, then if $\mu(\mathcal{F}) > \mu(\mathcal{E})$, any map

$$\mathcal{F} \to \mathcal{E}$$

must be zero ($\mu =$ slope).

Use this to deduce that stable bundles must be indecomposible (inexpressible as a direct sum of two non-trivial subbundles).

2. Show the holomorphic sections of the canonical line bundle on $\mathbb{C}P^\infty$ over an open subset U can be canonically identified with holomorphic functions on $\mathbb{C}P^1$ which vanish at the point $[0 : 1]$ (in other words, the sheaf of sections of this bundle is $\mathcal{O}(-1)$). Deduce that the canonical line bundle has no global holomorphic sections (this is probably the simplest example of a degree -1 line bundle having no holomorphic sections).

3. In class I asserted:

Proposition 0.2. For X a CW complex there is a natural isomorphism

$$[X, K(A, n)] \cong H^n(X; A).$$

I then stated that it followed that natural transformations of functors:

$$H^n(-; A_1) \to H^m(-; A_2)$$

were in bijective correspondence with $H^m(K(A_1, n); A_2)$. This is the Yoneda lemma. If you are unfamiliar with this fundamental lemma in category theory, look it up and convince yourself this is the case (it turns out to be a simple formal consequence).

We know that $H^*(K(\mathbb{Z}, 2), \mathbb{Z})$ is $\mathbb{Z}[x]$. What natural transformation does the element x^n correspond to?

4. Show that $\mathbb{R}P^2$ is not null cobordant.