The Energy and Rate Meta Distributions in Wirelessly Powered D2D Networks

Na Deng, *Member, IEEE* and Martin Haenggi, *Fellow, IEEE*

***Abstract*—As a key enabling technology for truly sustainable operation of devices, wireless energy and information transfer (WEIT) has attracted significant attention in wireless communication networks. Previous works on WEIT network analysis mostly concentrated on the energy outage probability or the expectation of the transferred energy at the typical wirelessly powered device using stochastic geometry. These calculations are relatively straightforward, but only provide limited information on the energy extracted by the individual devices. This paper considers a WEIT-enabled D2D network with the ambient RF transmitters distributed according to a Poisson point process and focuses on the *meta distribution* of the transferred energy, which is the distribution of the conditional energy outage probability given the locations of the RF transmitters, to show what fraction of devices in the network satisfy the target energy outage constraint if the required transmission energy is given. Furthermore, we derive the meta distribution of the transmission rate under an energy outage constraint and introduce a new notion of transmission efficiency, termed *wirelessly powered spatial transmission efficiency* (WP-STE), which is defined as the density of concurrently active links that rely on the wireless energy transfer technique and satisfy a certain reliability constraint that has a rate greater than a predefined threshold. Our analysis provides insightful guidelines for the most efficient way to operate a WEIT-enabled self-sustainable D2D communication network.**

***Index Terms*—Wireless energy and information transfer, wireless energy transfer, D2D communication, stochastic geometry, meta distribution.**

I. Introduction

A. Motivation

Device-to-device (D2D) communication has recently received great attention for its attractive traffic offloading capabilities and support of various location-based and peer-to-peer applications and services [1–3]. Most works on D2D communication assume that users have infinite battery capacity and are by definition willing to use their own power for data transmissions of others. However, due to the battery limitations, this assumption does not hold in practice and frequent battery replacement or recharging is often costly and even infeasible sometimes, which limits the advantages of D2D communications and their further developments. Therefore, efforts on prolonging the lifetime of D2D transmitters should be made to overcome this bottleneck.

Na Deng is with the School of Information and Communication Engineering, Dalian University of Technology (DLUT), Dalian, 116024, China (e-mail: dengna@dlut.edu.cn). Martin Haenggi is with the Dept. of Electrical Engineering, University of Notre Dame, Notre Dame 46556, USA (e-mail: mhaenggi@nd.edu).

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Recently, wireless energy transfer (WET) has emerged as a key candidate technology for future energy constrained wireless communication networks [4–7]. The integration of WET with communication networks, namely wireless energy and information transfer (WEIT)[1], holds the promise of many types of advantages [5]: powering wireless devices with continuous and stable energy over the air, improving user experience and convenience, allowing for high and sustainable throughput performance with low maintenance cost as well as enhanced flexibility in practical deployments, etc. Hence, WEIT in D2D communications can be a promising enabling solution and of fundamental importance to cope with the limited battery issue and facilitating the more widespread use of D2D communication. However, due to the distinct evaluation criteria and sensitivity level between the energy transfer and D2D transmission [4], the transmit power requirements and hence the transmission distances can be rather different. For instance, the efficiency of the energy transfer highly depends on the total received signal power while the reliability of D2D transmission is determined by the received signal-to-interference-plus-noise ratio (SINR). When stochastic geometry is used for the analysis, the energy transfer performance is most commonly evaluated at the typical link. While this performance is certainly important, it does not reveal how concentrated the energy transfer reliabilities of all the wirelessly powered devices are due to the spatial averaging involved (averaging over all links). To overcome this limitation, our focus in this paper is to quantify the variability of the energy transfer reliabilities around the expected value over the point process, i.e., to provide a refined analysis of the wirelessly transferred energy and the achievable rate under an energy outage constraint in a WEIT-enabled D2D network, and hence explore in depth how to exploit the RF signals to build a self-sustainable D2D network.

B. Related Work

As WET becomes increasingly feasible due to the reduction in power requirement of electronics and smart devices, many research efforts have advanced the theoretical understanding of wirelessly powered systems (see [5–7] for a comprehensive overview). The Poisson point process (PPP) has been by far the most popular spatial model for various types of wireless networks with RF transmitters and their corresponding performance evaluations due to its several convenient features, such

[1]This technique is sometimes also called wireless information and power transfer (WIPT).

as the independence between different points and the simple form of the probability generating functional (PGFL) [8–14].

Specifically, [8] introduced the concept of power beacons (PBs) as dedicated power sources to charge mobile devices and investigated the tradeoffs among transmit power and device density under an outage constraint in cellular networks, where the locations of base stations (BSs) and PBs are modeled as two independent homogeneous PPPs. As an extension, both [9] and [10] investigated the performance in heterogeneous cellular networks. Moreover, research efforts have also investigated WET in relay network [11] and cognitive radio network [12] using PPP-based models, respectively. As for WEIT-enabled D2D networks, a limited amount of research has been conducted: the study in [13] investigated cognitive D2D communication powered by the ambient interference from the overlaid cellular networks and derived the transmission probability and outage probability for both D2D transmitters and cellular users. In addition, [14] investigated secure D2D communication in cognitive cellular networks, where the D2D transmitter harvests energy from PBs. The energy transfer reliability and the secrecy performance were analyzed in a stochastic geometry framework.

Although the existing works have comprehensively investigated the wireless energy transfer technology in different scenarios, they characterized the energy and information transfer performance from an average perspective, i.e., merely focusing on the performance evaluated at the typical link, which, however, provides quite limited information on that of individual links. Very recently, there have been some works considering fine-grained performance metrics on the basis of the meta distribution [15] in conventionally powered wireless networks. The meta distribution was formally defined in [15] to provide a much sharper version of the "SIR performance" than that merely considered at the typical link through spatial averaging in two basic Poisson network models. From then on, it was analyzed in Poisson cellular networks with underlaying D2D communication in [16], with power control in [17], and with BS cooperation in [18], respectively. However, to our best knowledge, there is no work that has applied the concept of the meta distribution to wireless energy and information transfer and unraveled how the energy transfer and information transmission phases influence each other in terms of the fine-grained performance, which is a critical and unique problem in WEIT-enabled networks. In this work, we will fill this gap with new analytical results on the performance of both the wirelessly transferred energy and the achievable rate powered by the RF signals.

C. Contributions

The main objectives of this paper are to introduce and promote the meta distribution as a key performance metric for WEIT-enabled D2D networks and to analyze the meta distributions of the transferred energy and the transmission rate. The distinguishing features of wirelessly powered network from a conventional communication network motivate us to carry out a comprehensive investigation on the performance of the network powered by RF transmitters, aiming at finding the most efficient operating regime of WEIT in D2D networks. The contributions of this paper are summarized as follows:

- We first present a general framework for a comprehensive performance analysis in WEIT-enabled D2D networks using tools from stochastic geometry, where the D2D transmitters are first powered by randomly-located RF transmitters and then perform data transmission to their corresponding receivers. To the best of our knowledge, this is the first application of the meta distribution concept to WEIT-enabled networks.
- In the energy transfer phase, we propose three WET policies: 1) omnidirectional energy transfer (OET), where RF transmitters transfer energy through omnidirectional antennas; 2) nearest directed energy transfer (NDET), where the nearest RF transmitter transfers energy through a directional antenna; 3) paired directed energy transfer (PDET), where an RF transmitter is selected and paired with a powered device to transfer energy through a directional antenna. Based on the three policies, we derive lower bounds for the moments of the conditional energy outage probability given the point process. Then we give analytical expressions to bound and approximate the meta distribution of the harvested energy. It is demonstrated that the derived results provide asymptotic bounds and accurate approximations.
- In the information transfer phase, we study the quality of service (QoS) metric of D2D communications, i.e., the meta distribution of the transmission rate under an energy outage constraint which means a D2D transmitter is active only if it has sufficient energy. In addition to the link-level performance, we further introduce a new notion of transmission efficiency from a network view, named *wirelessly powered spatial transmission efficiency* (WP-STE), which is defined as the density of concurrent active RF-powered links that satisfy a certain reliability constraint and have a rate greater than a predefined threshold.
- Comparing the three WET policies: OET, NDET and PDET, we demonstrate the substantial benefits of using beamforming technique in WEIT in terms of higher energy transfer efficiency. Moreover, the impacts of some key parameters, such as the RF transmitter density, the transmit power of RF and D2D transmitters, etc., on each performance metric are investigated numerically, which sheds insightful tradeoffs in energy and information transmissions as well as guidelines for WEIT-enabled system design.

II. System Model

A. Network Model

We consider a D2D communication network powered solely by ambient RF transmitters (including cellular base stations, smart phones, digital TV towers, WiFi hotspots, etc.), where the D2D transmitters are distributed according to a homogeneous PPP Φ_{d} with density λ_{d}. Each D2D transmitter is assumed to be battery-less and utilize the instantaneously harvested RF energy to supply its operation[2], and to have a dedicated receiver at distance $r_{\rm d}$ in a random orientation, i.e., the D2D users form a Poisson bipolar network [20, Def. 5.8]. Each D2D user is equipped with a single antenna and transmits data with a constant power $\mu_{\rm d}$. The RF transmitters are modeled as an independent homogeneous PPP $\Phi_{\rm p}$ with density $\lambda_{\rm p}$ and transmit power $\mu_{\rm p}$. We assume that in each time slot, RF transmitters in $\Phi_{\rm p}$ independently transmit with probability p.

The channel (power) gain between transmitter x and receiver y is given by $h_{xy}\ell(x-y)$ where h_{xy} models the small-scale fading and $\ell(x-y)$ represents the large-scale path loss. We assume that all fading coefficients are i.i.d. exponential with unit mean (Rayleigh fading) for both information and energy transfer links, and $\ell(x)=\|x\|^{-\alpha}$, where α is the path loss exponent. We assume different path loss exponents for the information and energy transfer, denoted as $\alpha_{\rm d}$ and $\alpha_{\rm p}$, respectively.

B. Wireless Energy Transfer Model

We consider a simple yet effective energy transfer model. It is assumed that D2D transmitters adopt a time-switched "harvest-then-transmit" strategy, and RF transmitters use frequencies outside the data band (e.g., in the ISM band) and hence cause no interference to D2D transmission. Specifically, in each time slot, each D2D transmitter first uses a fraction η of the time slot to harvest energy from RF transmitters and then transmit the information to its corresponding receiver during the remaining $1-\eta$ fraction of time if the harvested energy satisfies the minimum requirement for signal transmission. This procedure means that all the harvested energy during the energy transfer time slot is used to transmit the information signals in the current information transmission duration, and there is no battery to store the remaining energy for future use as in [21]. We consider an additional D2D transmitter at the origin that attempts to operate D2D transmission using the energy harvested from the ambient RF transmitters. Due to Slivnyak's theorem [20, Thm. 8.10], this transmitter becomes the typical transmitter under expectation over the PPP.

1) Omnidirectional energy transfer (OET): In this case, each RF transmitter is equipped with an omni antenna, and D2D transmitters harvest their aggregate received energy transmitted by all active RF transmitters. This policy provides isotropic energy transfer to the devices while its transmission region is quite limited due to the severe propagation loss. Mathematically, by introducing the coefficient $\frac{\nu}{1+F}$ in [22], which is used to capture the randomness in the detection of the actual harvested energy, in each time slot, the harvested energy can be quantified as

$$\varepsilon_{\rm H-OET}=\frac{\nu\eta\rho}{1+F}\sum_{y\in\Phi_{\rm p}}\mu_{\rm p}h_y\ell(y)B_y, \tag{1}$$

where F follows an exponential distribution with parameter ζ, which is assumed to be fixed in each time slot and independent of $\Phi_{\rm p}$, $\Phi_{\rm d}$ and the channel fading coefficients, and accordingly, ν is chosen so that $\frac{\nu}{1+F}$ has an expectation of 1, i.e., $\nu=\frac{1}{-\zeta e^{\zeta}\,{\rm Ei}(-\zeta)}$, where Ei is the exponential integral function defined by ${\rm Ei}(x)=-\int_{-x}^{\infty}e^{-t}/t{\rm d}t$. ρ is the efficiency of the conversion from RF to DC power, and B_y is a Bernoulli variable with parameter p to indicate whether y is active. Without loss of generality, the duration of a time slot is set to one.

[2]The battery-free design has smaller size and simpler power management system and thus lends itself, for example, to lower-power medical devices [19].

TABLE I. Antenna parameters of a uniform linear antenna array [24]

Parameters	Description	value
w	Half-power beamwidth	$2\arcsin\left(\frac{2.782}{\pi N}\right)$
$\bar{G}_{\rm m}$	Main lobe gain	N
$\bar{G}_{\rm s}$	Side lobe gain	$1/\sin^2\left(\frac{3\pi}{2N}\right)$

2) Nearest directed energy transfer (NDET): In this case, each D2D transmitter is powered by its nearest RF transmitter and each RF transmitter is equipped with a directional antenna array synthesizing its highly directional beam pointing to its powered device. If several (at least two) D2D transmitters have the same nearest RF transmitter, multi-user beamforming techniques could be adopted or the beam direction could be pointed to different D2D transmitters in a time-division manner. Since the substantial array gains provided by the directional antenna array help to compensate for the propagation loss, the power transmission efficiency can be significantly improved, hence enlarging the energy transfer region. To maintain analytical tractability, a sectorized antenna model [23] is adopted to approximate the actual antenna pattern, formulated as

$$G(\varphi)=\begin{cases} G_{\rm m}=g\bar{G}_{\rm m} & \text{if } |\varphi|\le w/2\\ G_{\rm s}=g\bar{G}_{\rm s} & \text{otherwise,}\end{cases} \tag{2}$$

where $w\in(0,2\pi]$ is the half-power beam width and correlated with the size of antenna array, $\varphi\in[-\pi,\pi)$ is the angle off the boresight direction, $\bar{G}_{\rm m}$ and $\bar{G}_{\rm s}$ are the non-normalized array gains of the main and side lobes, and g is a scaling factor that ensures the power constraint $\frac{1}{2\pi}\int_0^{2\pi}G(\varphi){\rm d}\varphi=1$. With the assumption of a uniform linear antenna array with half-wavelength antenna spacing, the half-power beamwidth w, main lobe gain $\bar{G}_{\rm m}$, and side lobe gain $\bar{G}_{\rm s}$ for the antenna number $N\ge 2$ are summarized in Table I. Since the RF transmitters steer their beams toward their intended D2D transmitters, the beams from all the other energy transfer links (i.e., non-intended RF transmitters) are uniformly distributed in $[0,2\pi)$. As a result, the antenna gain for all non-intended RF transmitters is equal to $G_{\rm m}$ with probability $w_{\rm m}=w/(2\pi)$ and $G_{\rm s}$ with probability $w_{\rm s}=1-w_{\rm m}$.

Therefore, in each time slot, the harvested energy of the typical D2D transmitter is given by

$$\begin{aligned}\varepsilon_{\rm H-NDET} \;=\; & \frac{\nu\eta\rho}{1+F}\Bigg(\mu_{\rm p}G_{\rm m}h_{y_0}\ell(y_0)\\ & +\sum_{y\in\Phi_{\rm p}\setminus\{y_0\}}\mu_{\rm p}G(\varphi_y)h_y\ell(y)B_y\Bigg),\end{aligned} \tag{3}$$

where $y_0\in\Phi_{\rm p}$ represents the nearest RF transmitter and $G(\varphi_y)$ is given in (2).

3) Paired directed energy transfer (PDET): In this case, we consider the scenario that each D2D transmitter selects an RF transmitter nearby equipped with a directional antenna array through a handshaking protocol. The motivation behind the proposed PDET is that sometimes the nearest RF transmitter may be unavailable and the D2D transmitter has to find an available RF transmitter nearby to form an energy transfer link using beamforming[3]. We focus on the typical D2D transmitter that attempts to harvest energy from an additional RF transmitter y_0 synthesizing its highly directional beam pointing to the powered device. Due to the conditioning property of the PPP [20, Box 8.2], adding a point at y_0 is the same as conditioning on $y_0 \in \Phi_{\rm p}$ in a PPP. So, the statistics of the resulting total received power correspond to those conditioning on $y_0 \in \Phi_{\rm p}$. Since the homogeneous PPP is rotationally invariant, we assume $y_0 = (r_0, 0)$ with $r_0 = \|y_0\|$ to be the designated RF transmitter for the typical D2D transmitter without loss of generality.

Thus, in each time slot, the harvested energy of the typical D2D transmitter is given by

$$\varepsilon_{\rm H-PDET} = \frac{\nu\eta\rho}{1+F}\bigg(\mu_{\rm p} G_{\rm m} h_{y_0}\ell(y_0) + \sum_{y\in\Phi_{\rm p}} \mu_{\rm p} G(\varphi_y) h_y \ell(y) B_y\bigg), \tag{4}$$

where $G(\varphi_y)$ is given in (2). The main difference between NDET and PDET is that the additional RF transmitter y_0 is not necessarily the nearest RF transmitter to the typical D2D transmitter in the PDET policy.

C. Information Signal Model

We consider a D2D receiver at the origin that attempts to receive from an additional transmitter x_0 located at $(r_{\rm d}, 0)$. Due to Slivnyak's theorem, this receiver becomes the typical receiver under expectation over the PPP. Denoting $\mu_{\rm d}$ by the D2D transmit power, the D2D transmitters become active and transmit data if $\varepsilon_{\rm H} \geq (1-\eta)\mu_{\rm d}$. In other words, the active probability of D2D transmitters in $\Phi_{\rm d}$ is the probability that the harvested energy satisfies the constraint $\varepsilon_{\rm H}(x_0) \geq (1-\eta)\mu_{\rm d}$. When the typical link is active[4], the SIR at the typical receiver is given by

$$\mathrm{SIR} = \frac{\ell(x_0) h_{x_0}}{\sum_{x\in\Phi_{\rm d}} \ell(x) h_x \mathbf{1}(\varepsilon_{\rm H}(x) \geq (1-\eta)\mu_{\rm d})}. \tag{5}$$

where $\mathbf{1}(\varepsilon_{\rm H}(x) \geq (1-\eta)\mu_{\rm d})$ indicates whether the harvested energy is sufficient for the data transmission. Accordingly, the transmission rate at the typical receiver is given by

$$C = (1-\eta) W \log_2(1+\mathrm{SIR}), \tag{6}$$

where W is the bandwidth for D2D communications.

[3]The search process may be performed with the aid of the cellular base stations, which can gather the location information of the RF and D2D transmitters

[4]The received signal power is assumed zero if the desired transmitter is not active, so the SIR is zero in this case.

D. Performance Metrics

In this section, according to the concept of the meta distribution in [15], we formally introduce the meta distribution of the harvested energy and transmission data rate as fine-grained performance metrics of the WEIT system.

Definition 1. (Energy Meta Distribution) *The meta distribution of the harvested energy is the distribution of the link energy outage probability conditioned on the locations of the RF transmitters, defined as*

$$\bar{F}_{P_{\rm o}(\xi)}(x) \triangleq \mathbb{P}(P_{\rm o}(\xi) > x), \quad \xi \in \mathbb{R}^+, \quad x \in [0,1], \tag{7}$$

where $P_{\rm o}(\xi)$ is a random variable given as

$$P_{\rm o}(\xi) \triangleq \mathbb{P}^o(\varepsilon_{\rm H} < \xi \mid \Phi_{\rm p}). \tag{8}$$

$\bar{F}_{P_{\rm o}(\xi)}(x)$ represents the complementary cumulative distribution function (CCDF) of $P_{\rm o}(\xi)$. Here $P_{\rm o}(\xi)$ represents the conditional energy outage probability, i.e., the harvested energy at the origin is less than the energy threshold ξ conditioned on the locations of the RF transmitters and the typical D2D harvester at the origin o. Due to the ergodicity of the point processes, the meta distribution can be interpreted as the fraction of links in each realization of the point processes that have a harvested energy less than ξ with probability at least x. The standard energy outage probability is the mean of $P_{\rm o}(\xi)$, obtained by integrating the meta distribution (7) over $x \in [0,1]$.

According to the information signal model, the active probability of each D2D transmitter depends on whether the harvested energy is greater than its energy consumption for D2D transmission, which establishes an important relationship between the energy and information transfer. Based on this, we introduce the meta distribution of the transmission rate under an energy outage constraint as follows.

Definition 2. (Transmission Rate Meta Distribution) *The meta distribution of the transmission rate is the two-parameter distribution function*

$$\bar{F}_{P_{\rm s}(\tau)}(x) \triangleq \mathbb{P}(P_{\rm s}(\tau) > x), \quad \tau \in \mathbb{R}^+, \quad x \in [0,1]. \tag{9}$$

Here the conditional probability $P_{\rm s}(\tau)$ is the random variable

$$P_{\rm s}(\tau) \triangleq \mathbb{P}^o(C > \tau \mid \Phi_{\rm d}), \tag{10}$$

which is the conditional success probability corresponding to the case that the transmission rate at the origin exceeds the rate threshold τ conditioned on the locations of D2D transmitter and the typical D2D receiver at the origin.

Since a direct calculation of the meta distribution seems infeasible, we will derive an exact analytical expression through the moments $N_b(\xi) \triangleq \mathbb{E}\left[P_{\rm o}(\xi)^b\right]$ and $M_b(\tau) \triangleq \mathbb{E}\left[P_{\rm s}(\tau)^b\right]$ in the following analysis.

III. Meta Distribution of the Harvested Energy

In this section, for each of the three proposed WET policies, we provide the moments of the conditional energy outage probability given the point process of RF transmitters through deriving a lower bound, which are then used to give a very accurate approximation for the meta distribution of the harvested energy. Both the bound and approximation are proven to be very accurate with well controlled and mathematically quantified gaps.

A. Analysis of OET

In this policy, RF transmitters are equipped with omni antennas to transfer energy to D2D devices. Let $\delta_{\rm p} \triangleq 2/\alpha_{\rm p}$, $\varpi \triangleq \pi\lambda_{\rm p}\frac{\pi\delta_{\rm p}}{\sin(\pi\delta_{\rm p})}(\zeta\nu\eta\rho\mu_{\rm p})^{\delta_{\rm p}}$ and

$$\mathcal{D}_b(p,\delta_{\rm p}) \triangleq pb\,{}_2F_1(1-b,1-\delta_{\rm p};2;p),\ b\in\mathbb{C} \text{ and } p,\delta_{\rm p}\in[0,1], \tag{11}$$

where ${}_2F_1(\cdot)$ is the Gaussian hypergeometric function.

Theorem 1. (Moments for OET) *The moments $N_b(\xi)$, $b>0$ of the conditional energy outage probability for OET are lower bounded by*

$$\check{N}_b(\xi) = \exp\Big(-\varpi\xi^{-\delta_{\rm p}}\mathcal{D}_b(p,\delta_{\rm p})\Big). \tag{12}$$

Proof: See Appendix A.

Based on the lower bound $\check{N}_b(\xi)$ and the Gil-Pelaez theorem [25] with the imaginary moments $\check{N}_{jt}$ of $P_{\rm o}(\xi)$, $t\in\mathbb{R}$, $j\triangleq\sqrt{-1}$, we give an interval in which the exact meta distribution lies.

Theorem 2. (Bounds on energy meta distribution for OET) *The distribution of the conditional energy outage probability for OET is in the following interval:*

$$\bar{F}_{\rm H-OET}(x,\xi) \le \bar{F}_{P_{\rm o}(\xi)}(x) \le \bar{F}_{\rm H-OET}\big([x-1+e^{-\zeta}]^+,\xi\big), \tag{13}$$

where $[x]^+=\max\{x,0\}$ and $\bar{F}_{\rm H-OET}(x,\xi)$ is given by

$$\begin{aligned}\bar{F}_{\rm H-OET}(x,\xi) \;=\; &\frac{1}{2}-\frac{1}{\pi}\int_0^\infty \frac{\exp\big(-\varpi\xi^{-\delta_{\rm p}}\Re(\mathcal{D}_{jt})\big)}{t}\\ &\times\sin\big(t\log x+\varpi\xi^{-\delta_{\rm p}}\Im(\mathcal{D}_{jt})\big)\,{\rm d}t,\end{aligned} \tag{14}$$

where $\mathcal{D}_{jt}=\mathcal{D}_{jt}(p,\delta_{\rm p})$ is given in (11), $\Re(z)$ and $\Im(z)$ denote the real and imaginary parts of $z\in\mathbb{C}$.

Proof: Since it is difficult to directly obtain an explicit expression of the $P_o(\xi)$ and its corresponding distribution, we turn to deriving a bound $\check{P}_o(\xi)$ and the distribution of the bound. From the proof of Thm. 1, letting $Y=\nu\eta\rho/\xi\sum_{y\in\Phi_{\rm p}}\mu_{\rm p}h_y\ell(y)B_y$, we have

$$\begin{aligned}P_{\rm o}(\xi)-\check{P}_{\rm o}(\xi) &= \mathbb{P}(F>Y-1)-\mathbb{P}(F>Y)\\ &= \mathbb{E}_X\Big[\big(e^{-\zeta(Y-1)}-e^{-\zeta X}\big)\mathbf{1}_{Y\ge1}\\ &\quad+\big(1-e^{-\zeta Y}\big)\mathbf{1}_{Y<1}\Big]\\ &\le \mathbb{E}_Y\Big[\big(1-e^{-\zeta}\big)\mathbf{1}_{Y\ge1}+\big(1-e^{-\zeta}\big)\mathbf{1}_{Y<1}\Big]\\ &= 1-e^{-\zeta}.\end{aligned} \tag{15}$$

It follows that $\mathbb{P}(P_{\rm o}(\xi)>x)\le\mathbb{P}(\check{P}_{\rm o}(\xi)+1-e^{-\zeta}>x)$, and thus $\bar{F}_{P_{\rm o}(\xi)}(x)\le\bar{F}_{\check{P}_{\rm o}(\xi)}([x-1+e^{-\zeta}]^+)$, where $[x]^+=\max\{x,0\}$. Since $P_{\rm o}(\xi)\ge\check{P}_{\rm o}(\xi)$, we also have $\bar{F}_{P_{\rm o}(\xi)}(x)\ge\bar{F}_{\check{P}_{\rm o}(\xi)}(x)$. According to the Gil-Pelaez theorem, the CCDF of $\check{P}_{\rm o}(\xi)$ is given by

$$\bar{F}_{\check{P}_{\rm o}(\xi)}(x) = \frac{1}{2}+\frac{1}{\pi}\int_0^\infty\frac{\Im\left(e^{-jt\log x}\check{N}_{jt}\right)}{t}{\rm d}t, \tag{16}$$

where $\check{N}_{jt}$, $t\in\mathbb{R}$ is given in (12), and $\Im(z)$ is the imaginary part of $z\in\mathbb{C}$. Letting $\mathcal{D}_{jt}=\mathcal{D}_b(p,\delta_{\rm p})$, we have

$$\begin{aligned}\bar{F}_{\check{P}_{\rm o}(\xi)}(x) \;&=\; \frac{1}{2}+\frac{1}{\pi}\int_0^\infty\frac{\Im\left(e^{-jt\log x}\exp(-\varpi\xi^{-\delta_{\rm p}}\mathcal{D}_{jt})\right)}{t}{\rm d}t\\ &= \frac{1}{2}-\frac{1}{\pi}\int_0^\infty\frac{\exp\big(-\varpi\xi^{-\delta_{\rm p}}\Re(\mathcal{D}_{jt})\big)}{t}\\ &\quad\times\sin\big(t\log x+\varpi\xi^{-\delta_{\rm p}}\Im(\mathcal{D}_{jt})\big)\,{\rm d}t.\end{aligned} \tag{17}$$

■

Note that efficient calculation methods for the meta distribution have been proposed in [26, 27], which significantly reduce the computational complexity. An interesting question is how close the bounds are compared with the exact results. For $\zeta\to0$, it is answered in the following corollary.

Corollary 1. (Asymptotic behavior as $\zeta\to0$) *When $\zeta\to0$, $\bar{F}_{\rm H-OET}(x,\xi)\to\bar{F}_{P_{\rm o}(\xi)}(x)$ and $\check{N}_b(\xi)\to N_b(\xi)$ for $b>0$. Furthermore, $\bar{F}_{P_{\rm o}(\xi)}(x)$ and $N_b(\xi)$ are asymptotically lower bounded by $\bar{F}_{\rm H-OET}(x,\xi)$ and $\check{N}_b(\xi)$, i.e., $\bar{F}_{P_{\rm o}(\xi)}(x)\gtrsim\bar{F}_{\rm H-OET}(x,\xi)$ and $N_b(\xi)\gtrsim\check{N}_b(\xi)$, respectively.*

Proof: As $\zeta\to0$, we have $[x-1+e^{-\zeta}]^+\to x$, and

$$\bar{F}_{\rm H-OET}\big([x-1+e^{-\zeta}]^+,\xi\big)\to\bar{F}_{\rm H-OET}(x,\xi). \tag{18}$$

Due to the squeeze theorem [28], $\bar{F}_{P_{\rm o}(\xi)}(x)\to\bar{F}_{\rm H-OET}(x,\xi)$, as $\zeta\to0$. Moreover, we have

$$\bar{F}_{P_{\rm o}(\xi)}(x)\gtrsim\bar{F}_{\rm H-OET}(x,\xi), \tag{19}$$

where '$\gtrsim$' stands for an asymptotic lower bound, i.e., $\exists t>0$ s.t. $\bar{F}_{P_{\rm o}(\xi)}(x)>\bar{F}_{\rm H-OET}(x,\xi)\ \forall\zeta<t$. Since $N_b=\int_0^1 bt^{b-1}\bar{F}_{P_{\rm o}(\xi)}(t){\rm d}t$, we have $\check{N}_b(\xi)\to N_b(\xi)$ and $N_b(\xi)\gtrsim\check{N}_b(\xi)$, for $b>0$. ■

Cor. 1 implies that for small ζ, $\bar{F}_{\rm H-OET}(x,\xi)$ and $\check{N}_b(\xi)$ provide extremely accurate approximations to the meta distribution and the corresponding moments of the harvested energy, respectively. Furthermore, we investigate the variance of $P_{\rm o}(\xi)$ and its maximum to mathematically quantify the impacts of network parameters on the concentration of the conditional energy outage probability around the standard (mean) energy outage probability.

Corollary 2. *The maximum of* ${\rm var}\,P_{\rm o}(\xi)$ *for OET with respect to ξ only depends on the active probability and path loss exponent of RF transmitters, i.e., p and $\alpha_{\rm p}$, and is independent of all the other parameters.*

Proof: For small ζ, $N_b(\xi)\approx\check{N}_b(\xi)$, we obtain

$$\begin{aligned}{\rm var}P_{\rm o}(\xi)\;&\approx\;\check{N}_2(\xi)-\check{N}_1^2(\xi)\\ &=\;\exp\big(-\varpi\xi^{-\delta_{\rm p}}D_2\big)-\exp\big(-2\varpi\xi^{-\delta_{\rm p}}D_1\big),\end{aligned} \tag{20}$$

and its first derivative

$$\frac{{\rm d}{\rm var}P_{\rm o}(\xi)}{{\rm d}\xi}\;\approx\;\varpi\delta_{\rm p}\xi^{-\delta_{\rm p}-1}D_2\exp\big(-\varpi\xi^{-\delta_{\rm p}}D_2\big)$$

$$-2\varpi\delta_{\rm p}\xi^{-\delta_{\rm p}-1}D_1\exp\left(-2\varpi\xi^{-\delta_{\rm p}}D_1\right), \quad (21)$$

where $D_1 = \mathcal{D}_1(p,\delta_{\rm p})$ and $D_2 = \mathcal{D}_2(p,\delta_{\rm p})$. Letting $\frac{{\rm d}\,{\rm var}P_{\rm o}(\xi)}{{\rm d}\xi} = 0$, the unique extreme point is derived as

$$\xi^* = \Big(\frac{\varpi(2D_1-D_2)}{\log(2D_1/D_2)}\Big)^{1/\delta_{\rm p}}, \quad (22)$$

and it is easy to verify that ξ^* corresponds to the maximal value of ${\rm var}P_{\rm o}$, given by

$$\max{\rm var}P_{\rm o}(\xi) \;=\; \exp\left(-\frac{D_2\log(2D_1/D_2)}{2D_1-D_2}\right) - \exp\left(-\frac{2D_1\log(2D_1/D_2)}{2D_1-D_2}\right), \quad (23)$$

which only depends on p and $\alpha_{\rm p}$, and does not depend on the other parameters such as $\lambda_{\rm p}$, ζ, ν, η, ρ and $\mu_{\rm p}$. ■

Though the expression in Thm. 2 can be calculated via numerical integration techniques, it is difficult to gain insights directly and apply it to obtain other analytical results. To further simplify the calculation of the meta distribution, we use the approximation approach proposed in [29] by matching the first three moments of the generalized beta distribution with $\check{N}_n(\xi)$, $n=1,2,3$ given in Thm. 1. The probability density function (PDF) of a generalized beta distributed random variable X with parameters (κ,β,υ) is given by

$$f_X(x) = \frac{x^{\kappa-1}(1-x/\upsilon)^{\beta-1}}{\upsilon^{\kappa}B(\kappa,\beta)}\mathbf{1}_{x\le\upsilon}, \quad (24)$$

where $\upsilon\in(0,\,1]$ and $B(\cdot,\cdot)$ is the beta function. For $\upsilon=1$, this is the standard beta approximation. The parameters can be obtained through the moment matching method, given as

$$\mathbb{E}X = \frac{\upsilon\kappa}{\kappa+\beta}, \quad \mathbb{E}(X^n) = \upsilon\frac{\kappa+n-1}{\kappa+\beta+n-1}\mathbb{E}(X^{n-1}), \quad (25)$$

and the solutions to the three equations, i.e., $n=1,2,3$ can be easily obtained via the **fsolve** function in Matlab 2014 (and later versions). Thus, the generalized beta approximation of the meta distribution is obtained by

$$\bar{F}_{P_{\rm o}(\xi)}(x) \approx (1-I_x(\kappa/\upsilon,\beta))\mathbf{1}_{x\le\upsilon}, \quad (26)$$

where $I_x(\kappa,\beta)$ is the regularized incomplete beta function. The generalized beta distribution can also be used to approximate the meta distributions of the harvested energy in NDET and PDET policies.

B. Analysis of NDET

In this policy, each D2D device is powered by RF transmitters equipped with directional antenna arrays among which the nearest RF transmitter synthesizes a highly directional beam pointing to the powered device. Letting $\psi_{\rm m}\triangleq G_{\rm m}\zeta\nu\eta\rho\mu_{\rm p}$ and $\psi_{\rm s}\triangleq G_{\rm s}\zeta\nu\eta\rho\mu_{\rm p}$, we have the following theorem.

Theorem 3. (Moments for NDET) *The moments $N_b(\xi)$, $b>0$ of the conditional energy outage probability for NDET are lower bounded by*

$$\check{N}_b(\xi) \;=\; \int_0^\infty\!\exp\Big(-b\log\big(1+\xi^{-1}\psi_{\rm m}(r/(\lambda_{\rm p}\pi))^{-1/\delta_{\rm p}}\big) - r\big(1+\mathcal{F}_b(p,\delta_{\rm p},\xi,\sqrt{r/(\lambda_{\rm p}\pi)})\big)\Big){\rm d}r, \quad (27)$$

where

$$\mathcal{F}_b(p,\delta_{\rm p},\xi,r) \;=\; -\delta_{\rm p}\sum_{k=1}^{\infty}\binom{b}{k}(-p^{-1}\xi)^{-k}\sum_{n=0}^{k}(w_{\rm s}\psi_{\rm s})^n (w_{\rm m}\psi_{\rm m})^{k-n}\frac{r^{-k\alpha_{\rm p}}}{k-\delta_{\rm p}}F_{n,k-n}(p,\delta_{\rm p},\xi,r), \quad (28)$$

and

$$F_{n,k-n}(p,\delta_{\rm p},\xi,r) = \tilde{F}\big(k-\delta_{\rm p},n,k-n,k-\delta_{\rm p}+1;-\frac{\psi_{\rm s}}{\xi r^{\alpha_{\rm p}}};-\frac{\psi_{\rm m}}{\xi r^{\alpha_{\rm p}}}\big). \quad (29)$$

Here $\tilde{F}_1(\cdot)$ is the hypergeometric function of two variables[5] *[30, Chap. 9.18].*

Proof: See Appendix B.

Since the numerical evaluation of the analytical expressions in Thm. 3 is complicated, we further give a simple upper bound for the moments of the conditional energy outage probability by considering that the device is constrained to harvest energy only from the nearest RF transmitter in the following corollary.

Corollary 3. *The moments $N_b(\xi)$, $b>0$ of the conditional energy outage probability for NDET can be upper bounded by $\tilde{N}_b(\xi)$ asymptotically with $\zeta\to 0$, where*

$$\tilde{N}_b(\xi) = \sum_{k=0}^{\infty}\binom{b+k-1}{k}\xi^{-k}\big(-\psi_{\rm m}(\lambda_{\rm p}\pi)^{\frac{\alpha_{\rm p}}{2}}\big)^k\Gamma\big(1-\frac{k\alpha_{\rm p}}{2}\big). \quad (30)$$

Proof: See Appendix C.

Similar to OET, the meta distribution of conditional energy outage probability for NDET is bounded as

$$\bar{F}_{\rm H-NDET}(x,\xi) \le \bar{F}_{P_{\rm o}(\xi)}(x) \le \bar{F}_{\rm H-NDET}\big([x-1+e^{-\zeta}]^+,\xi\big), \quad (31)$$

where $[x]^+ = \max\{x,0\}$, and

$$\bar{F}_{\rm H-NDET}(x,\xi) \;=\; \frac{1}{2}-\frac{1}{\pi}\int_0^\infty\int_0^\infty\frac{e^{-r\left(1+\Re(\mathcal{F}_{jt}(\xi,r))\right)}}{t} \times\sin\Big(t\log\big(x+x\xi^{-1}\xi_{\rm m}(r/(\lambda_{\rm p}\pi))^{-1/\delta_{\rm p}}\big) + r\Im(\mathcal{F}_{jt}(\xi,r))\Big){\rm d}r{\rm d}t, \quad (32)$$

where $\mathcal{F}_{jt}(\xi,r) = \mathcal{F}_{jt}(p,\delta_{\rm p},\xi,r)$ is given in (28).

C. Analysis of PDET

In this policy, RF transmitters are equipped with directional antenna arrays to transfer energy to D2D devices using beamforming. In particular, we consider that the typical D2D transmitter has a designated RF transmitter at $y_0=(r_0,0)$ steering its beam to the device. Letting $\psi\triangleq\pi\lambda_{\rm p}(G_{\rm m}\zeta\nu\eta\rho\mu_{\rm p})^{\delta_{\rm p}}$ and $\tilde{\psi}\triangleq r_0^{-\alpha_{\rm p}}(\frac{\psi}{\pi\lambda_{\rm p}})^{1/\delta_{\rm p}}$, we have the following theorem.

[5]The $\tilde{F}_1$ function is also called the Appell function and is implemented in the Wolfram Language as **AppellF1**$[a,b_1,b_2,c,x,y]$.

Theorem 4. (Moments for PDET) *The moments $N_b(\xi)$, $b > 0$ of the conditional energy outage probability for PDET are lower bounded by*

$$\check{N}_b(\xi) = \exp\Big(-b\log(1+\tilde{\psi}\xi^{-1}) - \psi\xi^{-\delta_{\rm p}}\mathcal{X}_b(p,\delta_{\rm p})\Big), \quad (33)$$

where

$$\mathcal{X}_b(p,\delta_{\rm p}) \triangleq \delta_{\rm p}\sum_{k=1}^{\infty}\binom{b}{k}(-1)^{k+1}p^k\sum_{n=0}^{k}w_{\rm s}^n w_{\rm m}^{k-n} B(\delta_{\rm p},k-\delta_{\rm p})\,{}_2F_1(n,\delta_{\rm p};k;1-G_{\rm m}/G_{\rm s}). \quad (34)$$

Proof: See Appendix D.

Similar to OET and NDET, the meta distribution of conditional energy outage probability for PDET is bounded as

$$\bar{F}_{\rm H-PDET}(x,\xi) \le \bar{F}_{P_{\rm o}(\xi)}(x) \le \bar{F}_{\rm H-PDET}\big([x-1+e^{-\zeta}]^+,\xi\big), \quad (35)$$

where $[x]^+ = \max\{x,0\}$ and $\bar{F}_{\rm H-PDET}(x,\xi)$ is given by

$$\bar{F}_{\rm H-PDET}(x,\xi) = \frac{1}{2} - \frac{1}{\pi}\int_0^{\infty}\frac{\exp\left(-\psi\xi^{-\delta_{\rm p}}\Re(\mathcal{X}_{jt})\right)}{t} \times \sin\Big(t\log\big(x+x\tilde{\psi}\xi^{-1}\big)+\psi\xi^{-\delta_{\rm p}}\Im(\mathcal{X}_{jt})\Big)\,{\rm d}t, \quad (36)$$

where $\mathcal{X}_{jt} = \mathcal{X}_{jt}(p,\delta_{\rm p})$ is given in (34).

IV. Meta Distribution of the Transmission Rate

In this section, we use the meta distribution of the transmission rate[6] to characterize the transmission effectiveness when WEIT is applied to D2D communications. This fine-grained performance metric provides the information about what fraction of active D2D links in each realization of the point process is fully powered by RF transmitters and has a transmission rate C greater than τ with probability at least x. Letting $q \triangleq \mathbb{P}(\varepsilon_{\rm H} \ge (1-\eta)\mu_{\rm d})$ denote the fraction of nodes that have harvested sufficient energy in the energy transfer phase and are active in the information transfer phase, we derive the moments of the conditional rate distribution given the locations of D2D transmitters as follows.

Theorem 5. (Moments of transmission rate) *Given that the typical D2D link is active, the moments M_b ($b \in \mathbb{C}$) of the conditional rate distribution are*

$$M_b = \exp\left(-2\pi\lambda_{\rm d}\frac{\pi\delta_{\rm d}\theta^{\delta_{\rm d}}}{\sin(\pi\delta_{\rm d})}qb\,{}_2F_1(1-b,1-\delta_{\rm d};2;q)\right), \quad (37)$$

where $\theta = \big(2^{\frac{\tau}{(1-\eta)W}}-1\big)r_{\rm d}^{\alpha_{\rm d}}$, $\delta_{\rm d} = 2/\alpha_{\rm d}$ and $q = 1-\check{N}_1\big((1-\eta)\mu_{\rm d}\big)$.

Proof: Since the D2D transmitters become active and perform data transmission only if $\varepsilon_{\rm H} \ge (1-\eta)\mu_{\rm D}$, the active probability of D2D transmitters is approximately obtained by $q = 1-\check{N}_1(\xi)$, where $\xi = (1-\eta)\mu_{\rm d}$. Given $\Phi_{\rm d}$, the transmission success probability is

$$\begin{aligned} P_{\rm s}(\tau) &= \mathbb{P}(C > \tau \mid \Phi_{\rm d}) \\ &= \mathbb{P}(\mathrm{SIR} > 2^{\frac{\tau}{(1-\eta)W}}-1 \mid \Phi_{\rm d}) \\ &= \prod_{x\in\Phi_{\rm d}}\left(\frac{q}{1+\theta\|x\|^{-\alpha_{\rm d}}}+1-q\right) \quad (38)\end{aligned}$$

where $\theta = \big(2^{\frac{\tau}{(1-\eta)W}}-1\big)r_{\rm d}^{\alpha_{\rm d}}$.

Thus, we have

$$\begin{aligned} M_b &= \mathbb{E}\Big[\prod_{x\in\Phi_{\rm d}}\left(\frac{q}{1+\theta\|x\|^{-\alpha_{\rm d}}}+1-q\right)^b\Big] \\ &= \exp\left(-2\pi\lambda_{\rm d}\int_0^{\infty}\Big[1-\left(\frac{q}{1+\theta r^{-\alpha_{\rm d}}}+1-q\right)^b\Big]r{\rm d}r\right) \\ &= \exp\left(-2\pi\lambda_{\rm d}\frac{\pi\delta_{\rm d}\theta^{\delta_{\rm d}}}{\sin(\pi\delta_{\rm d})}qb\,{}_2F_1(1-b,1-\delta_{\rm d};2;q)\right). \quad (39)\end{aligned}$$

■

Then, the transmission rate meta distribution, denoted as $\bar{F}_{P_s(\tau)}(x)$, can be obtained by the Gil-Pelaez theorem with the imaginary moments M_{jt} of $P_{\rm s}(\tau)$, given by

$$\bar{F}_{P_s(\tau)}(x) = \frac{1}{2} - \frac{1}{\pi}\int_0^{\infty}\frac{\exp\left(-\theta^{\delta_{\rm d}}\Re(\mathcal{C}_{jt})\right)}{t} \times \sin\left(t\log x + \theta^{\delta_{\rm d}}\Im(\mathcal{C}_{jt})\right){\rm d}t, \quad (40)$$

where $\mathcal{C}_{jt} = 2\pi\lambda_{\rm d}q\,jt\frac{\pi\delta_{\rm d}}{\sin(\pi\delta_{\rm d})}\,{}_2F_1(1-jt,1-\delta_{\rm d};2;q)$.

Note that the meta distribution provides a refined performance characterization of all links, i.e., the entire distribution of a random variable instead of just the mean. Next, we will introduce a new fine-grained performance metric from a network level perspective, termed *wirelessly powered spatial transmission efficiency* (WP-STE), defined as follows.

Definition 3. (Wirelessly Powered Spatial Transmission Efficiency, WP-STE) *For a stationary and ergodic point process model, the WP-STE is*

$$S(\tau,x) \triangleq \lambda_{\rm d}q\bar{F}_{P_s(\tau)}(x), \quad (41)$$

where $\tau \in \mathbb{R}^+$, $x \in (0,1)$, $\lambda_{\rm d} > 0$, and $q \in (0,1]$.

Thus WP-STE is the density of concurrently active links that rely on the WET technique and satisfy a certain reliability requirement x that has a transmission rate greater than a predefined threshold τ. It is similar to the area spectral efficiency in conventional communication networks but with a QoS constraint applied at each individual link rather than the typical link.

With the simple approximation of the generalized beta distribution, $S(\tau,x)$ is approximated as

$$S(\tau,x) \approx \lambda_{\rm d}q(1-I_x(\kappa/v,\beta))\mathbf{1}_{x\le v}. \quad (42)$$

V. Numerical Results

In this section, we validate and illustrate the expressions derived in the previous section through numerical simulations. The main symbols and parameters are summarized in Table II with default values in the simulations.

[6]The SIR is assumed zero if the desired transmitter did not harvest sufficient energy, thus the rate is zero in this case.

TABLE II. Symbols and descriptions

Symbol	Description	Default value
$\Phi_{\rm p}$, $\lambda_{\rm p}$	RF transmitters PPP and density	$\mathrm{N/A}, 1$
$\Phi_{\rm d}$, $\lambda_{\rm d}$	D2D transmitters PPP and density	$\mathrm{N/A}, 0.1$
$\mu_{\rm p}$, $\mu_{\rm d}$	The transmit power of RF and D2D transmitters	1, 0.5
p, q	The active probability of RF and D2D transmitters	$0.5, \mathrm{N/A}$
$\alpha_{\rm p}, \alpha_{\rm d}$	The path loss exponent of the energy/information link	$4, 4$
ξ, τ	The energy/transmission rate threshold	0.1, 10 Mbps
η	The portion of time in the energy transfer phase	0.5
ρ	The efficiency of the conversion from RF to DC power	0.3 [22]
ζ	The parameter for random effect in harvested energy phase	0.01 [22]
ν	The normalized factor for random effect in harvested energy phase	N/A
r_0	The energy transfer link distance for PDET	1
N	The number of antennas of the RF transmitters	8
$G_{\rm m}, G_{\rm s}$	The main/side lobe of the antenna pattern for directed WET	N/A
W	D2D transmission bandwidth	10 MHz
$r_{\rm d}$	The link distance between the D2D users	1
$\bar{F}_{P_{\rm o}(\xi)}(x), \bar{F}_{P_{\rm s}(\tau)}(x)$	The meta distribution of the harvested energy/transmission rate	N/A
N_b, M_b	The b-th moments of the conditional energy outage/success probability	N/A
$\check{N}_b$	The lower bound for b-th moments of the conditional energy outage probability	N/A

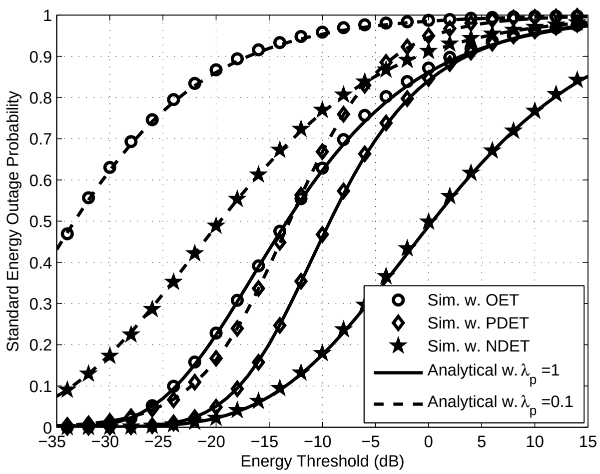

Fig. 1. The standard energy outage probability $\check{N}_1$ for different $\lambda_{\rm p}$.

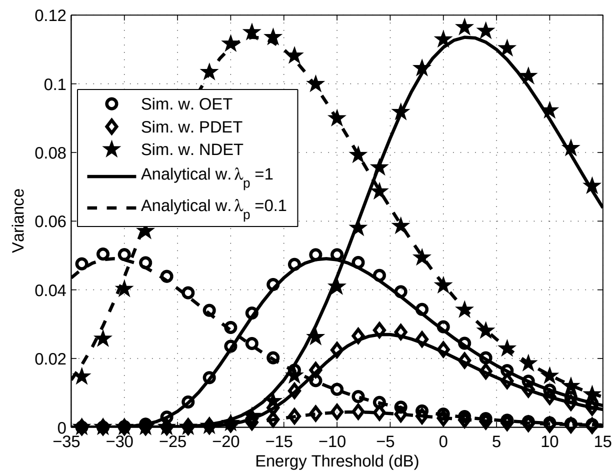

Fig. 2. The variance $\check{N}_2 - \check{N}_1^2$ of the conditional energy outage probability for different $\lambda_{\rm p}$.

A. The Energy Meta Distribution

Fig. 1 shows the standard energy outage probability (i.e., the first moment of $P_{\rm o}(\xi)$) versus the RF transmitter density $\lambda_{\rm p}$ using the lower bound $\check{N}_1$. It can be seen that the analytical results match accurately with the simulation results. As expected, given an energy threshold, a lower (average) energy outage probability can be achieved when $\lambda_{\rm p}$ is increased. The reason is straightforward: a larger $\lambda_{\rm p}$ implies that more RF transmitters are closer to the RF-powered device on average, thus resulting in lower energy outage probability. In addition, the better performance of PDET and NDET than that of OET lies in the high array gains of directional transmission, which significantly improves the energy transfer efficiency.

Fig. 2 presents the variance of the conditional energy outage probability as a function of ξ for different RF transmitter densities $\lambda_{\rm p}$ using the lower bounds $\check{N}_1$ and $\check{N}_2$. Since the variance necessarily tends to zero for both $\xi \to 0$ and $\xi \to \infty$, it assumes a maximum at some finite value of ξ. As shown in Cor. 2, it is seen that for different $\lambda_{\rm p}$, the maximal value of the variance in each WET policy is the same, i.e., it is not related to $\lambda_{\rm p}$. Moreover, the fluctuation of the variance for NDET is much more severe than that for OET and PDET, since in NDET the distance between an RF-power device and its nearest RF transmitter is a random variable compared with the fixed distance in PDET which has the smallest variance among the three WET policies.

As illustrated in Fig. 3, both the bounds (Thm. 2) and approximations (generalized beta distribution) provide an excellent match for the energy meta distribution, validating the analytical expressions derived in Sec. III. Additionally, the meta distribution shows qualitatively that, for the chosen parameters, 30% of the devices harvest an energy of 0.1 with outage probability greater than 80% for OET, while the same harvested energy is achieved with outage probability at least 80% by virtually no links for PDET and NDET. For quantitative purposes, the cross-sections are more informative.

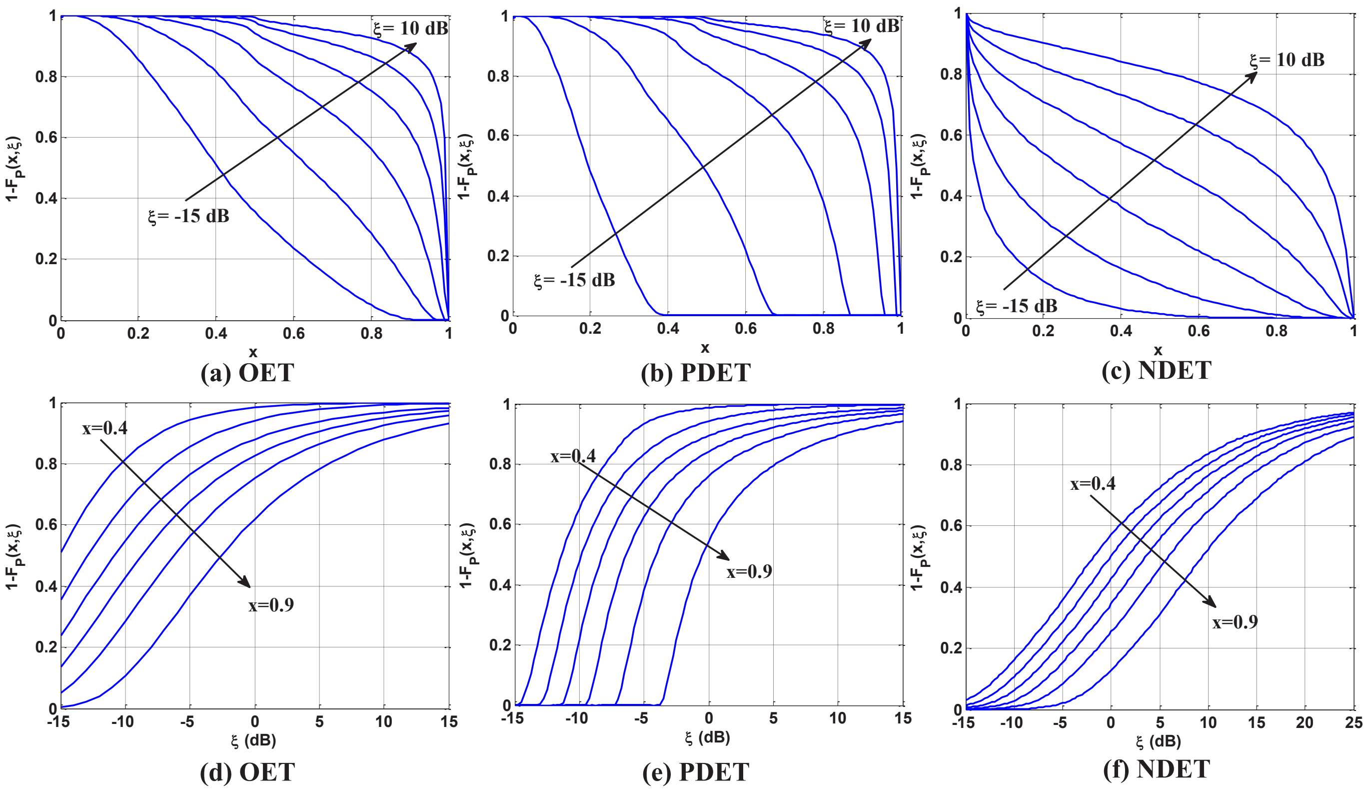

Fig. 4. Cross-sections through the meta distribution along the x and ξ axes.

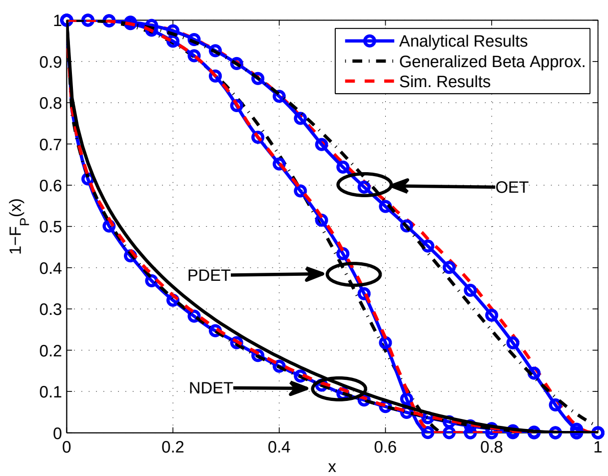

Fig. 3. The energy meta distributions for OET, PDET and NDET, respectively, where the solid line is the upper bound for NDET per (30).

Figs. 4(a)-(c) show the meta distributions of different energy thresholds for OET, PDET and NDET, respectively, which enable a more precise statement about what fraction of RF-powered devices harvest the required energy with a target reliability. For example, for OET and NDET, when $\xi = -5$ dB (0.3), about 75% of RF-powered devices have an energy outage probability of at least 60%, which indicates that 75% of devices have less than 40% possibility to harvest sufficient energy to operate the D2D transmission, while for PDET, only about 20% of devices that harvest an energy of 0.3 with the same energy transfer reliability.

As a function of ξ for fixed x, the value of ξ can be determined such that at least a fraction x of devices have a energy outage probability $p_{\min}$. For instance, Figs. 4(d)-(f) show that if a ξ of 0 dB is chosen, more than 60% and 50% of devices would suffer an energy outage probability of 90% for OET and PDET, respectively, while less than 15% of devices suffer the same outage probability for NDET. As seen from the plots, for the given system parameters, NDET using highly directional antennas brings huge benefits in terms of lower energy outage probability in dense deployment of RF transmitters ($\lambda_{\rm p} = 1$). For PDET, the performance depends highly on the fixed distance r_0 between an RF-powered device and its main RF transmitter which is set as twice as that for NDET on average. In PDET, the main RF transmitter is designated by the RF-powered device, and the distance depends on the selection. As expected, the shorter the distance, the higher the energy transfer efficiency of PDET.

B. Transmission Rate Meta Distribution

Fig. 5 compares the energy meta distribution for $\lambda_{\rm p} p = 0.25$ with different values of $\lambda_{\rm p}$ and p. As seen from the plots, the three curves for each WET policy have the same value of $\lambda_{\rm p} p$ and hence the same standard energy outage probability, but the corresponding meta distributions are rather different. This shows that the standard energy outage probability in conventional WET/WEIT works provides only limited information on the energy transfer performance.

In Fig. 6, we evaluate how the portion of energy transfer time in each time slot η influences the meta distribution of

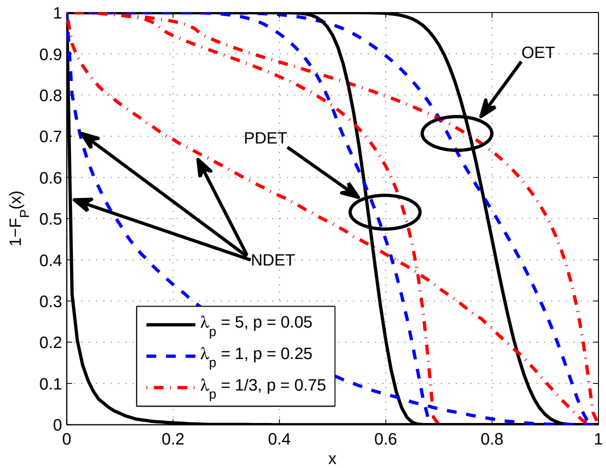

Fig. 5. The energy meta distribution vs. λ_{p} and p .

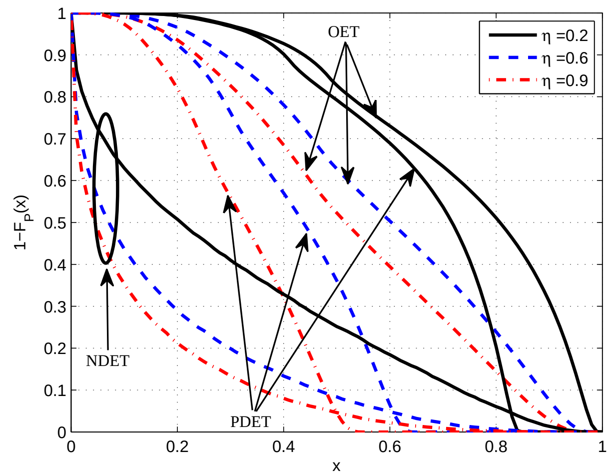

Fig. 6. The energy meta distribution vs. η.

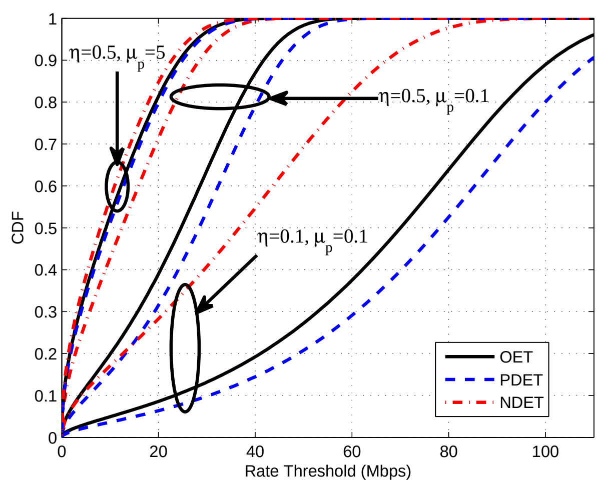

Fig. 7. The rate distribution for different η and μ_{p}.

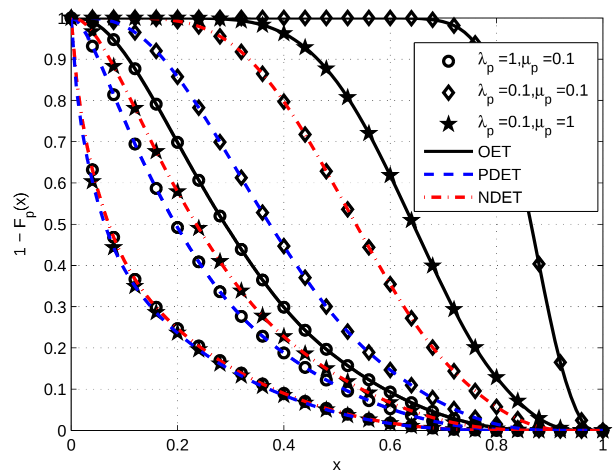

Fig. 8. The rate meta distribution for different λ_{p} and μ_p for $\mu_{\mathrm{d}} = 0.1$, $r_{\mathrm{d}} = 2$ and $\tau = 0.1$ Mbps.

harvested energy for the three proposed WET policies. It is seen that the energy transfer performance improves with the increase of η and when η is relatively small, the performance varies more dramatically with a change of η. In addition, the NDET is more robust to the varying of η than PDET and OET, since it has much higher energy transfer efficiency as shown in previous figures such that only a small portion of time is sufficient for the subsequent D2D transmission.

Fig. 7 demonstrates the impact of η and μ_p on the D2D transmission rate distribution. It is observed that for NDET, the transmit power of RF transmitters has a greater impact on the transmission rate than the portion of energy transfer time, while for PDET and OET, both parameters have significant impact on the rate performance. Moreover, there is no distinct performance disparity in the case of $\eta = 0.5$ and $\mu_{\mathrm{p}} = 5$, which is quite different in the other two cases. This implies a tradeoff between the link-level transmission rate and the energy transfer efficiency. When η and μ_p are relatively small, the proportion of the energy transfer is much smaller than that of the information transfer, and hence the network is energy-limited with high energy outage probability. As η and μ_p increase, the proportion of the energy transfer increases, making more and more D2D transmitters have enough energy for transmissions. At this time, however, the WEIT-enabled D2D network almost becomes a standard one which is interference-limited.

Fig. 8 shows the meta distribution of the transmission rate for different λ_{p} and μ_{p}. It is seen that OET always achieves higher transmission performance of D2D links than PDET and NDET, which is quite different from the results of energy meta distribution. The reason also lies in the tradeoff between energy harvesting and information transmission from a link-level view. Furthermore, for PDET, since increasing the transmit power of RF transmitters has a greater effect on decreasing the energy outage probability than increasing the RF transmitter density, the "$\star$" curve has a higher active probability of D2D transmitters corresponding to a lower transmission rate than the "$\circ$" curve relative to the "$\Diamond$" curve due to the stronger interference, while for NDET and OET, the results are just opposite.

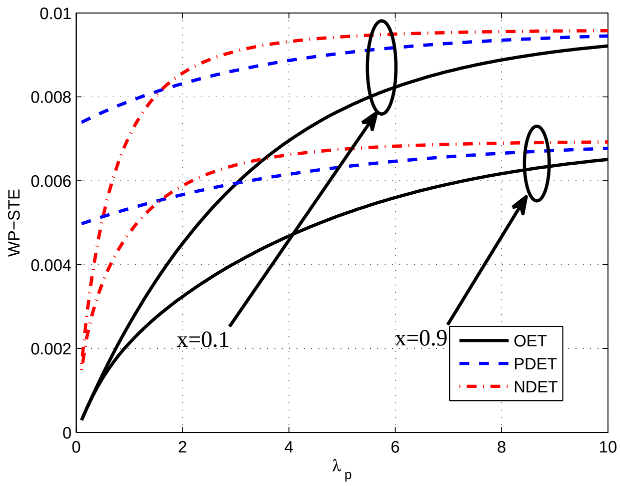

Fig. 9. The WP-STE vs. $\lambda_{\rm p}$ for $r_0 = 0.5$.

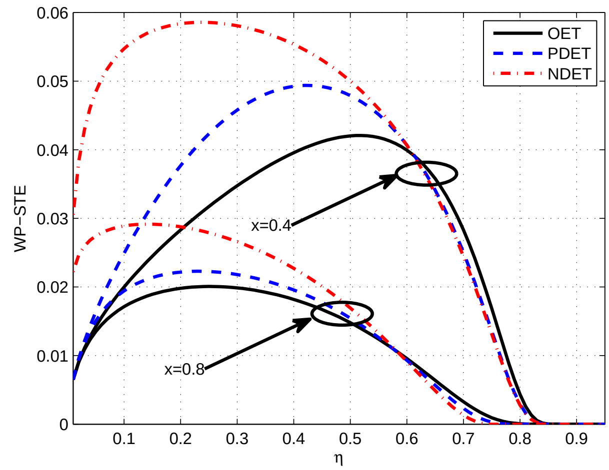

Fig. 10. The WP-STE vs. η for $r_0 = 1$.

C. WP-STE Analysis

Fig. 9 explores the behavior of $S(\tau, x)$ for fixed $\tau = 10$ Mbps and different x as a function of $\lambda_{\rm p}$. It is observed that PDET and NDET achieve higher spatial transmission efficiency than OET, demonstrating the benefit of using beamforming technique in WEIT system in terms of higher network-level performance. We also observe that the WP-STE increases with $\lambda_{\rm p}$ and gradually tends to be stable when the active probability of D2D transmitters tends to 1. In addition, the WP-STE of PDET is superior to NDET for small $\lambda_{\rm p}$, and the situation is reversed when $\lambda_{\rm p}$ is large. This implies that the RF transmitter density is closely related to the energy transfer distance that is of fundamental importance to achieve higher power and information transfer efficiency from a network point of view.

Fig. 10 investigates the influence of the energy transfer portion of time η on $S(\tau, x)$ for different x. It is seen that the curves with a lower target transmission reliability x have a better WP-STE since in this case more D2D links in the network achieve a rate greater than the threshold, thus resulting in a higher network performance. Moreover, the effect of η on $S(\tau, x)$ implies an important tradeoff between energy harvesting and information receiving in terms of the network-level performance WP-STE. It is straightforward that increasing η is beneficial for D2D transmitters to harvest more energy from the RF transmitters but the remaining time for the information transmission decreases instead. We can observe that $S(\tau, x)$ is a convex-like function of η. Specifically, when η varies from 0 to 1, $S(\tau, x)$ first increases and then decreases after reaching its maximum point. Additionally, the optimal value of η is dependent on the target transmission reliability and the adopted WET policy because the selection of η should not only ensure a sufficient energy transfer but also guarantee that the achievable transmission rate is greater than the predefined threshold, which is a crucial parameter for WEIT-enabled D2D networks.

VI. Conclusions

In this paper, we present a framework for a fine-grained analysis of WEIT-enabled D2D networks, where the meta distributions of the harvested energy and the transmission rate and the WP-STE are introduced as new performance metrics to characterize the performance of individual links or users in a given realization.

For the WET phase, we propose three energy transfer policies, namely, OET, NDET and PDET, and derive a lower bound for the moments of the conditional energy outage probability given the locations of the RF transmitters. On this basis, we provide tight bounds and a simple yet accurate approximation for the meta distribution of the harvested energy. Hence, the complete distribution of the conditional energy outage probability can be characterized, which provides much sharper results than merely the means (i.e., the average energy outage probability). The accuracy of the derived analytical expressions has been validated through numerical simulations. In particular, we observe that large density of RF transmitters and directional antenna arrays are important factors that lead to a better energy outage performance, while the maximum variance of the conditional energy outage probability merely depends on the active probability and path loss exponent of the RF transmitters.

For the wireless information transmission phase, we derive the meta distribution of the transmission rate as well as the WP-STE which gives a network-level performance metric based on the meta distribution with certain reliability constraint. The WEIT-enabled D2D network exhibits interesting tradeoffs between energy outage probability and transmission rate as well as the WP-STE, which are closely related to the proportion of energy and information transfer during the network operation. In other words, the most efficient operation regime of WEIT in D2D networks lies in the ability to balance the energy outage probability and the transmission reliability well and the knowledge about when the network is energy-limited, and when it is interference-limited.

In summary, the WEIT technique is expected to bring huge benefits for future wireless networks. However, some critical system parameters should be judiciously chosen in order to balance the inherent tradeoffs between the energy and information transfers. In addition, our analytical framework can be extended by considering other types of network where wireless transmission nodes rely on wireless charging to operate communications. As future works, a good direction is to generalize the results to other practical scenarios, such as considering other fading channels, non-linear energy harvesting models [31, 32], sophisticated power allocation schemes, rechargeable battery applications, etc. However, since the current results exploit the analytical tractability of Rayleigh fading and the linearity in the energy harvesting model, one might need to develop new analytical methods to cope with the more complicated cases.

APPENDIX A
PROOF OF THEOREM 1

Proof: Given $\Phi_{\rm p}$, the energy outage probability is

$$\begin{aligned} P_{\rm o}(\xi) &= \mathbb{P}\left(\frac{\nu\eta\rho}{F+1}\sum_{y\in\Phi_{\rm p}}\mu_{\rm p}h_y\ell(y)B_y < \xi \mid \Phi_{\rm p}\right) \\ &\geq \mathbb{P}\left(F > \frac{\nu\eta\rho}{\xi}\sum_{y\in\Phi_{\rm p}}\mu_{\rm p}h_y\ell(y)B_y \mid \Phi_{\rm p}\right) \\ &= \mathbb{E}\left[\exp\left(-\frac{\zeta\nu\eta\rho}{\xi}\sum_{y\in\Phi_{\rm p}}\mu_{\rm p}h_y\ell(y)B_y\right) \mid \Phi_{\rm p}\right] \\ &= \prod_{y\in\Phi_{\rm p}}\Big(\frac{p}{1+\bar{\xi}\|y\|^{-\alpha_{\rm p}}}+1-p\Big), \end{aligned} \tag{43}$$

where the last two steps follow from the exponential distributions of F and h_y, respectively, and $\bar{\xi} = \zeta\nu\eta\rho\mu_{\rm p}/\xi$. Letting

$$\check{P}_{\rm o}(\xi) = \prod_{y\in\Phi_{\rm p}}\Big(\frac{p}{1+\bar{\xi}\|y\|^{-\alpha_{\rm p}}}+1-p\Big), \tag{44}$$

we have $P_{\rm o}(\xi) \geq \check{P}_{\rm o}(\xi)$ and thus $\mathbb{E}[P_{\rm o}(\xi)^b] \geq \mathbb{E}[\check{P}_{\rm o}(\xi)^b]$, $b > 0$. The moments of $\check{P}_{\rm o}(\xi)$ are

$$\begin{aligned} \check{N}_b &= \mathbb{E}\Bigg[\prod_{y\in\Phi_{\rm p}}\Big(\frac{p}{1+\bar{\xi}\|y\|^{-\alpha_{\rm p}}}+1-p\Big)^b\Bigg] \\ &= \exp\left(-2\pi\lambda_{\rm p}\int_0^\infty\left[1-\Big(\frac{p}{1+\bar{\xi}r^{-\alpha_{\rm p}}}+1-p\Big)^b\right]r{\rm d}r\right) \\ &= \exp\left(-\pi\delta_{\rm p}\lambda_{\rm p}\int_0^\infty\sum_{k=1}^\infty\binom{b}{k}(-1)^{k+1}(p\bar{\xi})^k\frac{u^{\delta_{\rm p}-1}}{(u+\bar{\xi})^k}{\rm d}u\right) \\ &\overset{(a)}{=} \exp\left(-\pi\lambda_{\rm p}\frac{\pi\delta_{\rm p}}{\sin(\pi\delta_{\rm p})}\bar{\xi}^{\delta_{\rm p}}\sum_{k=1}^\infty\binom{b}{k}\binom{\delta_{\rm p}-1}{k-1}p^k\right) \\ &= \exp\left(-\pi\lambda_{\rm p}\frac{\pi\delta_{\rm p}\bar{\xi}^{\delta_{\rm p}}}{\sin(\pi\delta_{\rm p})}pb\,{}_2F_1(1-b,1-\delta_{\rm p};2;p)\right), \end{aligned} \tag{45}$$

where step (a) uses the following identity

$$\int_0^\infty\frac{u^{\delta_{\rm p}-1}}{(u+\bar{\xi})^k}{\rm d}u \equiv \frac{(-1)^{k+1}\pi}{\sin(\pi\delta_{\rm p})}\frac{\bar{\xi}^{\delta_{\rm p}-k}\Gamma(\delta_{\rm p})}{\Gamma(k)\Gamma(\delta_{\rm p}-k+1)}. \tag{46}$$

■

APPENDIX B
PROOF OF THEOREM 3

Proof: Given $\Phi_{\rm p}$ and denoting by y_0 the nearest point to the origin, the energy outage probability is

$$\begin{aligned} P_{\rm o}(\xi) &= \mathbb{P}\Bigg(\frac{\nu\eta\rho}{F+1}\Big(\mu_{\rm p}h_{y_0}\ell(y_0)G_{\rm m} \\ &\qquad + \sum_{y\in\Phi_{\rm p}\setminus\{y_0\}}\mu_{\rm p}h_y\ell(y)G(\varphi_y)B_y\Big) < \xi \mid \Phi_{\rm p}\Bigg) \\ &\geq \mathbb{E}\Bigg[\exp\Big(-\bar{\xi}\Big(h_{y_0}\ell(y_0)G_{\rm m} \\ &\qquad + \sum_{y\in\Phi_{\rm p}\setminus\{y_0\}}h_y\ell(y)G(\varphi_y)B_y\Big)\Big) \mid \Phi_{\rm p}\Bigg] \\ &= \frac{1}{1+G_{\rm m}\bar{\xi}\|y_0\|^{-\alpha_{\rm p}}}\prod_{y\in\Phi_{\rm p}\setminus\{y_0\}}\Big(\frac{w_{\rm m}p}{1+G_{\rm m}\bar{\xi}\|y\|^{-\alpha_{\rm p}}} \\ &\qquad + \frac{w_{\rm s}p}{1+G_{\rm s}\bar{\xi}\|y\|^{-\alpha_{\rm p}}}+1-p\Big). \end{aligned} \tag{47}$$

where $\bar{\xi} = \zeta\nu\eta\rho\mu_{\rm p}/\xi$. Letting $\xi_{\rm m} = \bar{\xi}G_{\rm m}$, $\xi_{\rm s} = \bar{\xi}G_{\rm s}$ and

$$\begin{aligned} \check{P}_{\rm o}(\xi) &= \frac{1}{1+\xi_{\rm m}\|y_0\|^{-\alpha_{\rm p}}}\prod_{y\in\Phi_{\rm p}\setminus\{y_0\}}\Big(\frac{w_{\rm m}p}{1+\xi_{\rm m}\|y\|^{-\alpha_{\rm p}}} \\ &\qquad + \frac{w_{\rm s}p}{1+\xi_{\rm s}\|y\|^{-\alpha_{\rm p}}}+1-p\Big), \end{aligned} \tag{48}$$

we have $P_{\rm o}(\xi) \geq \check{P}_{\rm o}(\xi)$ and thus $\mathbb{E}[P_{\rm o}(\xi)^b] \geq \mathbb{E}[\check{P}_{\rm o}(\xi)^b]$, $b > 0$. The moments of $\check{P}_{\rm o}(\xi)$ are

$$\begin{aligned} \check{N}_b &= \mathbb{E}\Bigg[\frac{1}{(1+\xi_{\rm m}\|y_0\|^{-\alpha_{\rm p}})^b}\prod_{y\in\Phi_{\rm p}\setminus\{y_0\}}\Big(\frac{w_{\rm m}p}{1+\xi_{\rm m}\|y\|^{-\alpha_{\rm p}}} \\ &\qquad + \frac{w_{\rm s}p}{1+\xi_{\rm s}\|y\|^{-\alpha_{\rm p}}}+1-p\Big)^b\Bigg] \\ &= \int_0^\infty f(r)e^{-b\log(1+\xi_{\rm m}r^{-\alpha_{\rm p}})}\exp\Bigg(-\pi\lambda_{\rm p} \\ &\qquad \underbrace{\times 2\int_r^\infty\left[1-\Big(\frac{w_{\rm m}p}{1+\xi_{\rm m}t^{-\alpha_{\rm p}}}+\frac{w_{\rm s}p}{1+\xi_{\rm s}t^{-\alpha_{\rm p}}}+1-p\Big)^b\right]t{\rm d}t}_{\mathcal{Y}(r)}\Bigg){\rm d}r, \end{aligned} \tag{49}$$

where $f(r) = 2\lambda_{\rm p}\pi re^{-\lambda_{\rm p}\pi r^2}$ is the distribution of $\|y_0\|$ [33], and $\mathcal{Y}(r)$ is obtained as

$$\begin{aligned} \mathcal{Y}(r) &= \sum_{k=1}^\infty\binom{b}{k}(-1)^{k+1}p^k\sum_{n=0}^k w_{\rm s}^n w_{\rm m}^{k-n} \\ &\qquad \times\int_{r^{\alpha_{\rm p}}}^\infty\frac{\delta_{\rm p}y^{\delta_{\rm p}-1}{\rm d}y}{(1+\xi_{\rm s}^{-1}y)^n(1+\xi_{\rm m}^{-1}y)^{k-n}} \\ &\overset{(a)}{=} \sum_{k=1}^\infty\binom{b}{k}(-1)^{k+1}p^k\sum_{n=0}^k(w_{\rm s}\xi_{\rm s})^n(w_{\rm m}\xi_{\rm m})^{k-n}\frac{r^{2-k\alpha_{\rm p}}\delta_{\rm p}}{k-\delta_{\rm p}} \end{aligned}$$

$$\times\tilde{F}(k-\delta_{\rm p},n,k-n,k-\delta_{\rm p}+1;-\frac{\xi_{\rm s}}{r^{\alpha_{\rm p}}};-\frac{\xi_{\rm m}}{r^{\alpha_{\rm p}}}),$$

where step (a) is obtained with the help of the formula in [30, Eq. 3.211]. The final result follows by substituting $\mathcal{Y}(r)$ into (30) and $\lambda_{\rm p}\pi r^2 \rightarrow r$. ■

APPENDIX C
PROOF OF COROLLARY 3

Proof: Given $\Phi_{\rm p}$ and denoting by y_0 the nearest point to the origin, the energy outage probability is

$$P_{\rm o}(\xi) < \mathbb{P}\left(\frac{\nu\eta\rho}{F+1}\mu_{\rm p}h_{y_0}\ell(y_0)G_{\rm m} < \xi \mid \Phi_{\rm p}\right). \quad (50)$$

Therefore, $P_{\rm o}(\xi)$ is upper bounded by the energy outage probability that the typical device merely harvests energy from y_0, and we have

$$\begin{aligned}\hat{P}_{\rm o}(\xi) &= \mathbb{P}\left(\frac{\nu\eta\rho}{F+1}\mu_{\rm p}h_{y_0}\ell(y_0)G_{\rm m} < \xi \mid \Phi_{\rm p}\right)\\ &\geq \mathbb{E}\Big[\exp\left(-\xi_{\rm m}h_{y_0}\ell(y_0)\right) \mid \Phi_{\rm p}\Big]\\ &= \frac{1}{1+\xi_{\rm m}\|y_0\|^{-\alpha_{\rm p}}}, \end{aligned} \quad (51)$$

where $\xi_{\rm m} = G_{\rm m}\zeta\nu\eta\rho\mu_{\rm p}/\xi$. Letting $\tilde{P}_{\rm o}(\xi) = \frac{1}{1+\xi_{\rm m}\|y_0\|^{-\alpha_{\rm p}}}$, we have $\hat{P}_{\rm o}(\xi) \geq \tilde{P}_{\rm o}(\xi)$ and thus $\mathbb{E}[\hat{P}_{\rm o}(\xi)^b] \geq \mathbb{E}[\tilde{P}_{\rm o}(\xi)^b]$, $b>0$. The moments of $\tilde{P}_{\rm o}(\xi)$ are

$$\begin{aligned}\tilde{N}_b &= \mathbb{E}\left[\frac{1}{(1+\xi_{\rm m}\|y_0\|^{-\alpha_{\rm p}})^b}\right]\\ &= \int_0^\infty 2\lambda_{\rm p}\pi r e^{-\lambda_{\rm p}\pi r^2}\frac{1}{(1+\xi_{\rm m}r^{-\alpha_{\rm p}})^b}{\rm d}r\\ &= \int_0^\infty 2\lambda_{\rm p}\pi r e^{-\lambda_{\rm p}\pi r^2}\sum_{k=0}^{\infty}\binom{b+k-1}{k}(-\xi_{\rm m})^k r^{-k\alpha_{\rm p}}{\rm d}r\\ &= \sum_{k=0}^{\infty}\binom{b+k-1}{k}\big(-\xi_{\rm m}(\lambda_{\rm p}\pi)^{\alpha_{\rm p}/2}\big)^k\Gamma\big(1-\frac{k\alpha_{\rm p}}{2}\big). \end{aligned} \quad (52)$$

As $\zeta \rightarrow 0$, $\tilde{N}_b \rightarrow \mathbb{E}[\hat{P}_{\rm o}(\xi)^b]$, and thus $\tilde{N}_b$ provides an asymptotic upper bound for N_b. ■

APPENDIX D
PROOF OF THEOREM 4

Proof: Given $\Phi_{\rm p}$ and an additional RF transmitter y_0 with $\|y_0\| = r_0$, we have

$$\begin{aligned}P_{\rm o}(\xi) &= \mathbb{P}\Bigg(\frac{\nu\eta\rho}{F+1}\Bigg(\mu_{\rm p}h_{y_0}\ell(y_0)G_{\rm m}\\ &\qquad +\sum_{y\in\Phi_{\rm p}}\mu_{\rm p}h_y\ell(y)G(\varphi_y)B_y\Bigg) < \xi \mid \Phi_{\rm p}\Bigg)\\ &\geq \frac{1}{1+G_{\rm m}\bar{\xi}\|y_0\|^{-\alpha_{\rm p}}}\prod_{y\in\Phi_{\rm p}}\Big(\frac{w_{\rm m}p}{1+G_{\rm m}\bar{\xi}\|y\|^{-\alpha_{\rm p}}}\\ &\qquad +\frac{w_{\rm s}p}{1+G_{\rm s}\bar{\xi}\|y\|^{-\alpha_{\rm p}}}+1-p\Big), \end{aligned} \quad (53)$$

where $\bar{\xi} = \zeta\nu\eta\rho\mu_{\rm p}/\xi$. Letting $\xi_{\rm m} = \bar{\xi}G_{\rm m}$, $\xi_{\rm s} = \bar{\xi}G_{\rm s}$ and

$$\begin{aligned}\check{P}_{\rm o}(\xi) &= \frac{1}{1+\xi_{\rm m}r_0^{-\alpha_{\rm p}}}\prod_{y\in\Phi_{\rm p}}\Big(\frac{w_{\rm m}p}{1+\xi_{\rm m}\|y\|^{-\alpha_{\rm p}}}\\ &\qquad +\frac{w_{\rm s}p}{1+\xi_{\rm s}\|y\|^{-\alpha_{\rm p}}}+1-p\Big), \end{aligned} \quad (54)$$

we have $P_{\rm o}(\xi) \geq \check{P}_{\rm o}(\xi)$ and thus $\mathbb{E}[P_{\rm o}(\xi)^b] \geq \mathbb{E}[\check{P}_{\rm o}(\xi)^b]$, $b>0$. The moments of $\check{P}_{\rm o}(\xi)$ are

$$\begin{aligned}\check{N}_b &= \frac{1}{\big(1+\xi_{\rm m}r_0^{-\alpha_{\rm p}}\big)^b}\mathbb{E}\Bigg[\prod_{y\in\Phi_{\rm p}}\Bigg(\frac{w_{\rm m}p}{1+\xi_{\rm m}\|y\|^{-\alpha_{\rm p}}}\\ &\qquad +\frac{w_{\rm s}p}{1+\xi_{\rm s}\|y\|^{-\alpha_{\rm p}}}+1-p\Bigg)^b\Bigg]\\ &= e^{-b\log(1+\xi_{\rm m}r_0^{-\alpha_{\rm p}})}\exp\Bigg(-\pi\lambda_{\rm p}\\ &\qquad \times\underbrace{2\int_0^\infty\Bigg[1-\Bigg(\frac{w_{\rm m}p}{1+\xi_{\rm m}r^{-\alpha_{\rm p}}}+\frac{w_{\rm s}p}{1+\xi_{\rm s}r^{-\alpha_{\rm p}}}+1-p\Bigg)^b\Bigg]r{\rm d}r}_{\mathcal{X}}\Bigg) \end{aligned} \quad (55)$$

$\mathcal{X}$ is obtained as

$$\begin{aligned}\mathcal{X} &= \sum_{k=1}^{\infty}\binom{b}{k}(-1)^{k+1}p^k\sum_{n=0}^{k}w_{\rm s}^n w_{\rm m}^{k-n}\\ &\qquad \times\int_0^\infty\frac{\delta_{\rm p}y^{\delta_{\rm p}-1}{\rm d}y}{(1+\xi_{\rm s}^{-1}y)^n(1+\xi_{\rm m}^{-1}y)^{k-n}}\\ &\overset{(a)}{=} \sum_{k=1}^{\infty}\binom{b}{k}(-1)^{k+1}p^k\sum_{n=0}^{k}w_{\rm s}^n w_{\rm m}^{k-n}\delta_{\rm p}\xi_{\rm m}^{\delta_{\rm p}}\\ &\qquad \times B(\delta_{\rm p},k-\delta_{\rm p})\,{}_2F_1(n,\delta_{\rm p};k;1-G_{\rm m}/G_{\rm s}), \end{aligned} \quad (56)$$

where step (a) is obtained with the help of [30, Eq. 3.197.1]. The final result is obtained by substituting $\mathcal{X}$ into (55). ■

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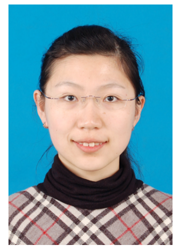

Na Deng (S'12-M'17) received the Ph.D. and B.S. degrees in electronic engineering from the University of Science and Technology of China (USTC), Hefei, China, in 2015 and 2010, respectively. From 2013 to 2014, she was a Visiting Student in Prof. Martin Haenggi's Group at the University of Notre Dame, Notre Dame, IN, USA. From June 2015 to November 2016, she was a Senior Engineer at Huawei Technologies Co., Ltd., Shanghai, China. Since then, she has been a lecturer with the School of Information and Communication Engineering, Dalian University of Technology, Dalian, China. Her scientific interests include networking and wireless communications, green communications, and network design based on wireless big data.

Martin Haenggi (S'95-M'99-SM'04-F'14) received the Dipl.-Ing. (M.Sc.) and Dr.sc.techn. (Ph.D.) degrees in electrical engineering from the Swiss Federal Institute of Technology in Zurich (ETH) in 1995 and 1999, respectively. Currently he is the Freimann Professor of Electrical Engineering and a Concurrent Professor of Applied and Computational Mathematics and Statistics at the University of Notre Dame, Indiana, USA. In 2007-2008, he was a visiting professor at the University of California at San Diego, and in 2014-2015 he was an Invited Professor at EPFL, Switzerland. He is a co-author of the monographs "Interference in Large Wireless Networks" (NOW Publishers, 2009) and "Stochastic Geometry Analysis of Cellular Networks (Cambridge University Press, 2018) and the author of the textbook "Stochastic Geometry for Wireless Networks" (Cambridge, 2012), and he published 15 single-author journal articles. His scientific interests lie in networking and wireless communications, with an emphasis on cellular, amorphous, ad hoc (including D2D and M2M), cognitive, and vehicular networks. He served as an Associate Editor of the Elsevier Journal of Ad Hoc Networks, the IEEE Transactions on Mobile Computing (TMC), the ACM Transactions on Sensor Networks, as a Guest Editor for the IEEE Journal on Selected Areas in Communications, the IEEE Transactions on Vehicular Technology, and the EURASIP Journal on Wireless Communications and Networking, as a Steering Committee member of the TMC, and as the Chair of the Executive Editorial Committee of the IEEE Transactions on Wireless Communications (TWC). Currently he is the Editor-in-Chief of the TWC. He also served as a Distinguished Lecturer for the IEEE Circuits and Systems Society, as a TPC Co-chair of the Communication Theory Symposium of the 2012 IEEE International Conference on Communications (ICC'12), of the 2014 International Conference on Wireless Communications and Signal Processing (WCSP'14), and the 2016 International Symposium on Wireless Personal Multimedia Communications (WPMC'16). For both his M.Sc. and Ph.D. theses, he was awarded the ETH medal. He also received a CAREER award from the U.S. National Science Foundation in 2005 and three awards from the IEEE Communications Society, the 2010 Best Tutorial Paper award, the 2017 Stephen O. Rice Prize paper award, and the 2017 Best Survey paper award, and he is a 2017 and 2018 Clarivate Analytics Highly Cited Researcher.