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Vehicular mobility patterns and their applications to Internet-of-Vehicles: a comprehensive survey

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Abstract With the growing popularity of the Internet-of-Vehicles (IoV), it is of pressing necessity to understand transportation traffic patterns and their impact on wireless network designs and operations. Vehicular mobility patterns and traffic models are the keys to assisting a wide range of analyses and simulations in these applications. This study surveys the status quo of vehicular mobility models, with a focus on recent advances in the last decade. To provide a comprehensive and systematic review, the study first puts forth a requirement-model-application framework in the IoV or general communication and transportation networks. Existing vehicular mobility models are categorized into vehicular distribution, vehicular traffic, and driving behavior models. Such categorization has a particular emphasis on the random patterns of vehicles in space, traffic flow models aligned to road maps, and individuals' driving behaviors (e.g., lane-changing and car-following). The different categories of the models are applied to various application scenarios, including underlying network connectivity analysis, off-line network optimization, online network functionality, and real-time autonomous driving. Finally, several important research opportunities arise and deserve continuing research efforts, such as holistic designs of deep learning platforms which take the model parameters of vehicular mobility as input features, qualification of vehicular mobility models in terms of representativeness and completeness, and new hybrid models incorporating different categories of vehicular mobility models to improve the representativeness and completeness.

Keywords vehicular mobility pattern, Internet-of-Vehicles (IoV), traffic flow, spatial point process, trajectory prediction, machine learning, deep learning

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1 Introduction

With the proliferation of wireless devices and technologies, things are ubiquitously connected to the Internet, sending data and requests and receiving services (e.g., navigation or location-based services) [1]. Increasingly equipped with wireless interfaces, vehicles including driver-less vehicles make up a significant part of this new paradigm [2]. As a consequence, not only do wireless networks grow quickly in terms of network scale, coverage and density, but become increasingly abundant in dynamics and variations such as fast-changing network topologies, channel states, and service demands [3]. Driven by big data, deep learning and deep reinforcement learning techniques have been applied to communication networks to optimize access, routing, resource allocation, and network security, after many successes in computer version and natural language processing [4]. It is anticipated that learning techniques will address many

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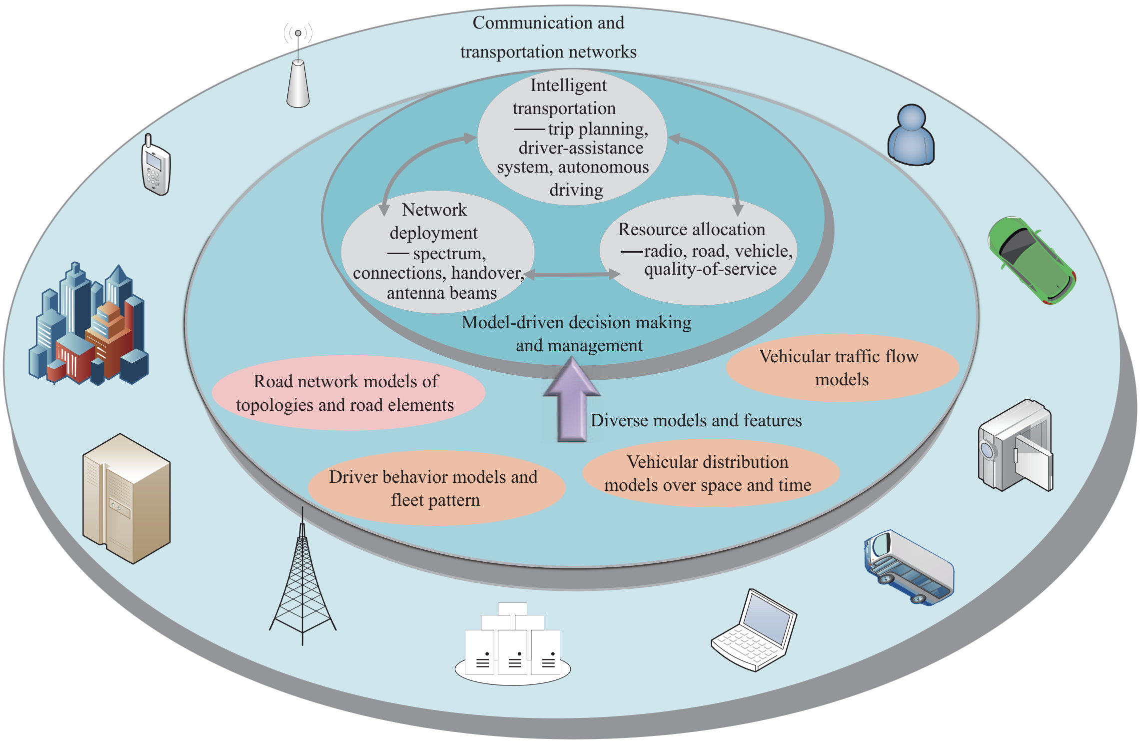


Figure 1 (Color online) The framework for model-driven applications in smart city (see Section 1).

challenging communication and networking problems which have not been adequately addressed using optimization techniques and analyses [1, 5].

An interesting area that deep (reinforcement) learning can revolutionize is network planning and optimization in future ubiquitous and fast-changing wireless networks [1]. In particular, the Internet-of-Vehicles (IoV), also known as vehicular ad-hoc networks (VANETs) or intelligent transport systems (ITS), are featured by large-scale networks and continuous changes in the networks, and strong coupling in both the time and space domains. Communication resources, such as radio resources for communication between vehicles and between vehicles and road-side infrastructures, networking resources, such as routing or handover decisions and connectivity, and security and privacy considerations all need to be meticulously optimized to achieve efficiency and accuracy in the immense variety of constantly changing network situations [5].

An application of learning techniques to the IoV is not trivial because of the aforementioned network scale, continuous network changes, and strong network coupling. In other words, data-driven deep learning techniques can potentially suffer from prohibitively large data sets, penalizing the effectiveness and efficiency of deep (reinforcement) learning [6]. Consider a real-time optimization for millions of vehicles in a typical medium to large city. There can be millions of inputs and millions of outputs of the learning, and they may need to be updated on a regular basis of seconds, if not milliseconds [7]. Feature extraction is an effective way to reduce this input, facilitate the data-driven learning process, and improve the efficiency [4, 7–9]. A large number of vehicular mobility models or patterns have been developed to extract characteristics and statistics of vehicles, which can potentially serve as useful features for data-driven deep (reinforcement) learning. The use of the models can improve the transferability and interoperability in the cases where distributed learning (such as federated learning) is carried out between different cities or countries, and the vehicular mobility models provide an efficient means to share the features, rather than huge amounts of raw data, between the cities. This is expected to significantly reduce the bandwidth requirements, improve the generalization of learning results, and protect privacy.

Accurate models of vehicular mobility have a wide range of applications for mobile communication networks, IoV, and ITS. Figure 1 provides a diagram for the application of vehicular mobility modeling in communication and transportation networks. There are three different stages in all of which the vehicular mobility models play an important role. First, accurate vehicular mobility models can help predict the demand for wireless resources, such as spectrum, connections, handover, and antenna beams,

Table 1 The comparison of different vehicular mobility prediction algorithms

Data set	Random forest	Adaboost	GBDT	SVM
Training data	0.96	0.96	0.94	0.84
Test data	0.83	0.82	0.82	0.71

over different space and time domains [2]. Second, the use of adequate vehicular mobility models allows for appropriate designs of radio resource allocation, networking, and quality-of-service (QoS) provision in real-time, or even in advance. Last but not least, the statistical knowledge obtained by accurate vehicular mobility models can help farsightedly optimize vehicular communication networks and roadside infrastructures for low cost and power consumption, high reliability and security, and high efficiency. Many existing studies are interested in vehicular mobility models [10–13]. For example, Refs. [11, 12] classified existing vehicle traffic models according to different levels of granularity. The vehicle mobility models are typically categorized into trip-, path- and flow-level motion models.

This survey reviews the status quo of vehicular mobility models, with a particular emphasis on the recent results in the past decade. We categorize the existing vehicular mobility models from the perspectives of modeling methodology, practicality, accuracy, complexity, and generality. Throughout this survey, we summarize the related application examples to cover a wide range of application scenarios in the ITS. We assess the practicality of the models in the actual context of the ITS and discuss the potential performance of existing models, according to the characteristics of different application scenarios. Despite several existing surveys [11–20], a comprehensive review and comparison study of vehicular models has yet to avail with extra attention to machine learning and big data analysis. For example, existing surveys, such as [14, 15], focus on the development status of the car-following models and lane-changing models, respectively.

The rest of this survey is organized as follows. In Section 2, we provide an overview of the vehicular mobility models and present the rationale and principles under our categorization of the models. From Section 3 to Section 5, we describe in detail the models of vehicle distributions over space and time, vehicular traffic flow, and drivers' behaviors and fleet patterns. The development of the road models is described in Section 6 to further study the communication aspect of ITS. In Section 7, taking into account actual situations, three specific classes of application of the models are described for joint network design, off-line network optimization, and real-time autonomous driving. In Section 8, a brief comparison of this survey to existing related surveys is carried out. Open challenges and conclusions are provided, respectively, in Sections 9 and 10.

2 An overview of vehicular mobility models

Data-driven machine learning, quickly gaining in popularity, has been increasingly applied to wireless communication to assist with network planning, operations, and optimization [21]. For example, based on the ETC data set and GPS data set of two million vehicles passing through in Guangdong Province, China for two months, Ref. [22] proposed a model to predict the destination, route, and speed of a single vehicle based on historical and real-time ETC data. The Mondrian forest model was used to integrate liquidity features and solve the uncertainty problem in liquidity prediction. We captured a large amount of GPS data of about 12509 taxis, spanning from November 1 to November 27, 2012 in Beijing. Data-driven machine learning can be useful to derive inference and prediction on the next moves of individual vehicles. There are five possibilities for a vehicle's possible next move: parking (or staying), going straight, left turn, right turn, and U-turn. We compared four classifiers for the vehicular mobility prediction. They are random forest (RF) [23], adaptive boosting (Adaboost) [24], gradient boosting decision tree (GBDT) [25], and Support Vector Machine (SVM) [26]. The classification accuracy of the four algorithms on the training data (which are the data captured from November 1 onwards) and the test data (which are the data captured from November 21 onwards) is shown in Table 1.

The RF, Adaboost, and GBDT can provide a prediction accuracy over 90%. The prediction can be helpful for wireless resource allocation, handover, dynamic network deployment, and help suggest routes or detours. Such learning has been based on huge amounts of explicit data which are typically not portable and can have specific features adhering to specific cities and environments, and may not be able to generalize and cause over-fitting. The classifiers can have an error of generalization, as shown to be the discrepancy between the test and training data sets in Table 1. New data-driven learning techniques, such

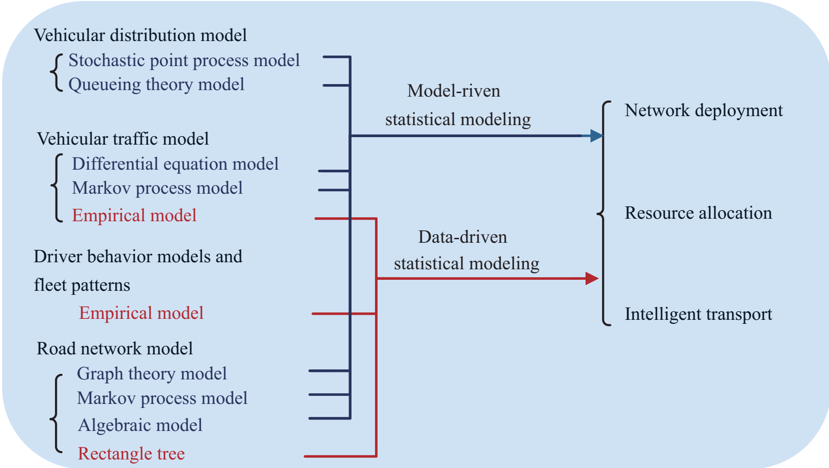


Figure 2 (Color online) Classification of models based on application scenes (see Section 2), the red font models are data-driven, the blue font models are model-driven, and the four types of models support three applications: network deployment, resource allocation, and intelligent transportation.

as transfer learning [27], or concept drift allows machine learning to start training from a trained machine learning model which is far more portable. There are also other new machine learning techniques, such as federated learning [28], where multiple learning agents can learn from their own data sets and synthesize on a regular basis for a generalized model. All this aspires to develop accurate vehicular models with specifics characterized for particular cities and regions, or drivers' behaviors. Encapsulating such models on different aspects of vehicular networks may inspire a new surge of holistic designs of communication mechanisms, infrastructure deployment, and governing policy and regulation.

Effective deployment of base stations (BSs) and roadside units (RSUs) is essential for efficient network planning and BS utilization [29]. Mobility model will greatly influence the performance of VANETs, Ref. [30] evaluated the impact of speed differences among vehicles over the performance of the furthest distance and link quality based schemes, which are two widely adopted classes of messaging schemes in VANETs. The influence of the vast speed differences between vehicles can bring obvious end-to-end delays to link quality based schemes. At present, stochastic geometry (SG) is most used to theoretically evaluate the performance of wireless networks. SG provides many fine-grained metrics, including reliability, throughput, delay, and their tradeoffs. Congestion, caused by the continuously increasing number of vehicles, adds higher demands to urban road system design [12, 31]. These demands include an effective and farsighted plan of urban space and a holistic assessment of network capacity. Vehicular traffic models assist in devising or optimizing resource allocation schemes [32]. Driver behavior models and fleet patterns formalize the complex relation between the driver and the traffic system [33]. Two corresponding application scenarios are forward-looking real-time route planning and danger warning, for example, real-time autonomous driving supported by the ultra-reliable and low-latency wireless network.

The purpose of this study is to provide a comprehensive review of the state-of-the-art vehicular mobility models, modeling methodologies, and their potential application scenarios. Meanwhile, the review shall help readers to identify suitable models, given application scenarios. The following quickly summarizes an overview of vehicular distribution models, vehicular traffic models, driving behavior models, and road network models. The classification of models based on application scenarios is visualized, which are divided into model-driven models and data-driven models, as shown in Figure 2. The model-driven modeling is to select a suitable one from existing models library based on intuition, personal experience, or formula derivation. The data-driven means that progress in modeling is compelled by data, rather than by intuition or by personal experience. Such models are especially effective if it is difficult to build model-driven simulation models (e.g., due to lack of understanding of the underlying processes), or the available models are not adequate [34].

2.1 Vehicular distribution model over space and time

Vehicular distribution models can be used to study the connectivity of vehicles and the connectivity between vehicles and roadside network infrastructures. The spatial distribution of vehicles is typically generated by using stochastic point processes, while queueing theory models are often used to describe the time-domain characteristics. Stochastic point process models provide a mathematical framework to describe the random location of vehicles in VANETs and reveal random spatial features of roads [35]. The stochastic point process models can be classified based on whether there is an interaction between the points or not [36]. Meta distribution is applied to the analysis of vehicle networks. Aggregated 1D homogeneous Poisson point process (PPP) is used to model the vehicles on the multi-lane highway. According to the meta distribution, a rate control scheme for per vehicle is proposed to keep all vehicles meeting the target link reliability [37]. It is vital to analyze the connectivity of VANETs for the deployment of base stations and other infrastructures.

2.2 Vehicular traffic flow model

Vehicular traffic flow models include broad aspects such as transportation planning, traffic flow, and traffic control. Vehicular traffic flow models describe the general laws which are observed and summarized from historical data in the long term [38]. According to the modeling methodologies, the models fall into three categories: continuous kinetic models based on differential equations, discrete grid models based on Markov processes, and empirical models. The analysis of the traffic load and the prediction of the travel demand is essential to the allocation of network resources.

2.3 Driver behavior model and fleet pattern

Driver behavior models and fleet patterns are of practical value to vehicle-to-vehicle (V2V) networks, which present three important features of vehicles, namely trajectory, car following, and lane changing. Driver behavior models describe drivers' maneuver decisions under varieties of traffic conditions. Fleet patterns capture the mobility of a group of vehicles, such as a platoon. The car-following model is used to determine how vehicles follow one another on a roadway [14]. The lane-changing model is captured by differential equations or drivers' lane-changing decision models [15]. The behavior of drivers is susceptible to the environment, road conditions, and their psychological and physical state [39,40]. The psychological state of drivers is hard to fully capture, which limits the accuracy of driving behavior models. The two existing methods are continuous Markov processes and the empirical model. The Markov processes are to describe the state transitions, such as the velocity, direction, and position. Artificial intelligence techniques, such as neural networks, have great potential to describe the psychological differences among drivers [15].

2.4 Road network model

Road topologies have a significant influence on the traffic flow pattern and spatial distribution of vehicles [41–43]. Large-scale road system models describe the topology of the road network and typically represent road systems as a directed graph [44] or a stochastic line process. Small-scale models are based on a high-precision map to generate simulated test scenarios. The small-scale models are capable of describing lane-level information such as curvature, position, and heading directions [45, 46]. They are divided into the Markov process (e.g., [47]), and more detailed graph-based models (e.g., [48]) such as rectangle-tree (R-tree) [45]. Ref. [49] utilized machine learning methods to predict average car velocities on selected streets, so it is crucial to map the notifications to the streets and lanes the cars come from. R-trees, standing for rectangular trees, turned out to be very well suited for solving this problem, which has important advantages in database applications, which can reduce the amount of hard drive searching. For example, the states of roads can be the states of their traffic lights. The transition of the road state can be modeled as a Markov process which can depend on the characteristics of the road. By taking into account the road topology in vehicular mobility models, the accuracy of models can be improved [41–43, 50, 51]. Integrating the information of lanes into a specific driver behavior model is the key to achieving autonomous driving in practice.

3 Vehicular distribution models over space and time

The vehicular arrival process at a road captures the time-domain features of the vehicular network and is typically modeled by applying queueing theory models. A widely adopted approach to model vehicle spatial distributions is to apply stochastic geometry, where random point processes are used to describe

Table 2 Characteristics and application of various stochastic geometry models (see Section 3)

Point process taxonomy	Interaction	Example	Application	Comparison
Poisson point process	Zero	Poisson point process	Character general characteristics of vehicular spatial distribution [52–59]	Simplicity and convenience
Cluster point process	Attraction	Cox process	Analyze coverage probability of typical receivers [29, 41, 42, 60, 61]	Capturing the time correlation of spatial distribution
Regular point process	Repulsion	Bernoulli lattice model	Network performance analysis [62]	Developed for network performance analysis
		Matérn hard-core process	Coverage-centered base station distribution [69]	
Queueing theory model	Zero	Poisson arrival location model	Calculate key quantities [63–68], as network connectivity, call density, and handoff rate	Describing the arrival process of vehicles

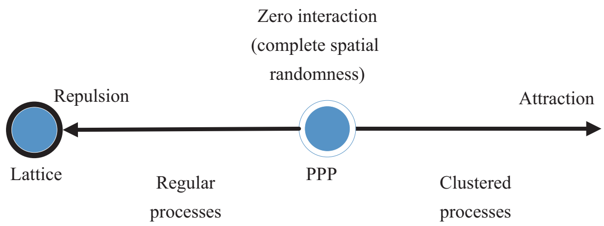


Figure 3 (Color online) Point process taxonomy (see Subsection 3.1 [36, Figure 3.1]). Upon moving to the left on this axis, the points begin to repel each other, which leads to more regular processes. In the most extreme case, the process becomes a lattice model. Upon moving to the right, the points attract each other, resulting in clustered processes.

the distributions of the location of vehicles in VANETs. The Poisson distribution is often used to indicate the time instants of the vehicles arriving at the road. This section describes the stochastic geometric-based models, which are popular in analyzing the throughput and connectivity of the network. Some application examples are shown in Table 2 [29, 41, 42, 52–69].

3.1 Motivation to extract distribution features

The convenience and efficiency of ITS highly depend on the reliable transmission of time-critical information to/among connected vehicles on roads. Implementing reliable ultra-low latency connection in a high-speed environment is important to the rollout of ITS [70]. The communication network of vehicles has the following characteristics, making it necessary to develop a suitable communication strategy for ITS [71].

- The vehicular movement in IoV is limited to a predetermined road system and road topology.
- The mobility of nodes is directly affected by the traffic density, which depends on road capacity and driver behavior.
- The communication environment between vehicles is complex and keeps changing. Network connectivity is affected by factors such as traffic conditions and vehicular mobility, which have a strong impact on the stability and efficiency of information transmission among communication nodes in VANETs.

The probability distribution of inter-vehicle spacing plays a crucial role in many connectivity studies [72]. The stochastic point process models enable an analysis of the distribution of vehicles and the connectivity of VANETs. By studying the spatial distributions of vehicles, researchers can optimize the network connection, quality of service, and deployment of charging stations and parking lots [73, 74].

The vehicle locations further depend on the geometry of the streets, road topologies, and buildings. There are morning and evening peaks in one day, and workdays and weekend days in one week. The spatial distribution of vehicles shows strong heterogeneity [41, 42, 75]. It exhibits clusters in some hot spots, such as commercial areas, hospitals, and schools.

From the simplest ideal infinite single lane to more sophisticated road conditions with traffic signal lights, intersections, multi-lanes, and other factors, steady progress has been made in the study of the spatial distribution of vehicles [62, 76–78]. However, there is no comprehensive survey in place, as shown in Section 8. We classify existing spatial distribution models based on whether the interaction between vehicles is captured. A taxonomy of the point process is shown in Figure 3 (see [36, Figure 3.1]). There is no interaction between points in the frequently-used PPP model.

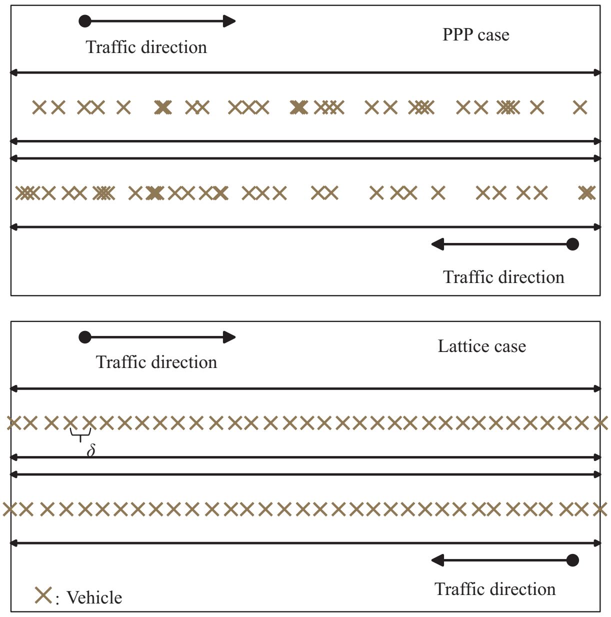


Figure 4 (Color online) The PPP model and the lattice model. The direction of the adjacent lanes is reversed. The δ of lattice model is the periodic locations in space (see Section 3).

3.2 Spatial stochastic process models without interaction

In the free-flow state, the position of the vehicles is completely random and independent of each other with a low traffic density. In this regime, the PPP is suitable because of its simplicity and tractability [78]. Refs. [76, 78, 79] verified the correctness of the Poisson hypothesis based on measured data under the free-flow condition. Compared with actual highway traffic data, the vehicle locations can be accurately approximated by a Poisson process especially in the case of sparse traffic, such as around midnight and early morning [78]. A stochastic geometric model is proposed to describe wireless network architectures [77]. A road system is modeled into a Manhattan Poisson line process (MPLP) in [80]. The locations of the vehicles on a certain lane are completely independent of each other in the PPP, as illustrated in Figure 4. The probability mass function (PMF) of the number of vehicles in the unit δ (length for one-dimensional and area for two-dimensional scenarios) is $P(N=k)=\frac{(\rho\delta)^k}{k!}\mathrm{e}^{-(\rho\delta)}$, where ρ is the intensity.

The above models are based on single-lane scenarios. The same method is also used on multi-lane highways. In [54], each lane is modeled as a one-dimensional PPP. Based on random geometry and queueing theory, the probability of transmission success is studied for inter-vehicle communication in a multi-lane highway. There are intersections in general IoV scenarios, and the two-dimensional model needs to be established. Some studies [81–83] focused on the intersections. They modeled the intersection as two roads perpendicular to each other, where vehicles on each road obey a PPP model.

For mathematical tractability, most existing studies assume that nodes are uniformly and randomly distributed in the region. The vehicles either are stationary, or completely disordered and independent of each other. This assumption is inaccurate to capture the spatial distribution of vehicles and their motion. The spatial distribution of vehicles is not completely random, showing location correlation [41]. Next we will introduce the vehicle distribution model in a general scenario. The PPP models are compared in Table 2.

Jeyaraj and Haenggi [43] analyzed a square (orthogonal) grid street system with Poisson distributed vehicles on each street. The transmitting vehicles on each street form an independent one-dimensional homogeneous PPP. Each transmitter has a dedicated recipient at a fixed distance. Exact analytical expressions for the success probabilities of the typical general/intersection users are derived, capturing the average performance of all the users [43]. In [84], the position of vehicles on the road is modeled as a spatial Poisson point process to characterize various aspects of the random behavior of vehicle-to-vehicle interference. Under the assumption that the vehicles equipped with communication devices follow a PPP, at any time instant, the vehicles on a particular road segment form a PPP, and thus the inter-vehicle spacing is exponentially distributed [52, 53, 55–59]. The behavior of V2V transmission with non-saturated data traffic is investigated using a continuous-time Markov chain model in [85]. The study in [85] combines queueing theory and point process theory. It reveals that vehicular transmitters can form a PPP collectively from the perspective of a static vehicular recipient.

We summarize the characteristics of PPP in Table 2. The computational convenience of the PPP model makes it a useful tool for describing network performance.

3.3 Spatial stochastic process model with inter-vehicle interaction

The distribution of vehicles displays a strong correlation in different cities [41, 42, 75], while the widely used PPP models assume that vehicles are completely independent. It is unable to accurately describe the vehicular distribution with a safe inter-vehicle distance. Therefore, the repulsion or attraction of points is introduced into the point process models, as shown in Figure 3.

3.3.1 *Clustered process*

Vehicles on typical roads tend towards clustering, due to traffic congestion and intersections [41,42]. Point processes with the attraction between points are proposed to more accurately describe the vehicular distribution than PPP. The typical attractive models are the Cox process model and other clustered models which are more irregular than the PPP. Ref. [86] introduced a model that takes the middle route between the complicated Cox vehicular network models and the oversimplified 2D PPP. The model called the transdimensional Poisson point process (TPPP) is the superposition of one or two 1D PPPs and a 2D PPP. The TPPP includes the 1D PPPs on the streets passing through the receiving vehicle and models the remaining vehicles as a 2D PPP ignoring their street geometry. It is shown that the TPPP provides good approximations to the more cumbrous models with streets characterized by Poisson line/stick processes. A comparison of clustered point processes is provided in Table 2.

Jeong et al. [73] modeled the random locations of vehicles as a stationary Cox process with Fox's H-distributed random intensity. Chetlur et al. [60] analyzed how the sparsity of vehicles and roads affects the coverage probability of a receiver. It is assumed that these receivers connect to the nearest node in networks using a slotted ALOHA channel access protocol. The concept of coverage probability is explained in [60]. The coverage probability is positively correlated with the vehicle density on roads, and negatively correlated with the road density [60]. A general framework for vehicle network modeling was proposed in [87], in which 1D PPP was formed independently for the position of vehicles on each street, and the spatial model of the whole framework was the Cox model.

In addition to V2V communication, there is another communication mode in IoV by deploying a roadside unit (RSU) to provide network access, real-time security information, and so on. A certain number of gateways, for instance, a cellular mobile communication network, are added to the VANETs to expand the V2V communication. Therefore, the distribution of the base stations of cellular networks is also an essential part of modeling and analyzing IoV. The Poisson cluster process [88–90] may be used to describe the BS location distribution. The authors of [61] modeled the spatial distribution of vehicles as a Gaussian process and the mobility demand pattern as a log-Gaussian process. Chetlur et al. [91] used Poisson line Cox process (PLCP) to model vehicles, 2D PPP to model the location of cellular Acer station (MBSs), and calculated PMF of tagged MBS loads serving typical vehicle users.

Cui et al. [41, 42] built a generic model which can describe the spatial point patterns of random vehicle locations in cities. A log-Gaussian Cox process (LGCP) model is proposed, by analyzing real location data of taxi trajectories in Beijing and Porto. The LGCP model can capture multiple point patterns of vehicles. The LGCP model can be used to study the connectivity and capacity of networks [41, 42]. Ref. [87] modeled and analyzed vehicular networks. Vehicles on each street form independent 1D Poisson point processes, and the street system specifies the random intensity measurement of a Cox process of vehicles. It is proved that the Cox vehicular networks behave like 2D PPP in the low-reliability regime. Some applications of the cluster processes are summarized in Table 2.

Remark. Compared with the PPP, the clustered point processes are a better fit for the spatial distribution of vehicles in a modern city. They are suitable for analyzing network capacity and connectivity.

3.3.2 *Regular process*

Regular processes, which are spatial point processes with repulsion between vehicles, are often used to describe point patterns with regular location distributions [92]. Regular processes exhibit a repulsive force between points and result in a minimum inter-vehicle distance. Vehicles are distributed according to a deterministic (regular) one-dimensional lattice in a lattice process, as illustrated in Figure 4 (see [62]). However, such a model tends to overestimate the connectivity of VANETs.

Guo et al. [93] used the coverage probability to evaluate the different point processes of describing the accuracy of a real base station deployment. The experiment fits the Strauss process (SP), the Poisson hard-core process (PHCP), and the perturbed triangular lattice by minimizing the gap between the coverage probability of the model and that of the real data. And the fitted models are close to the coverage probability of the real point sets. As revealed experimentally [89], the deployment of the base stations (BSs) is capacity-oriented in urban areas with high density and coverage-oriented in rural areas. The distribution of BSs in urban areas can be modeled as an attractive process, such as the Matérn cluster process [36, 89]. The distribution of BSs in rural areas can be modeled as a repulsive process, such as the Strauss hard-core process [36, 89]. The hard-core process helps analyze network connectivity for rural areas.

Determinantal point processes (DPPs) [92] are another class of models for capturing the inter-point repulsion. The Ginibre point process (GPP) [94] is a relatively tractable DPP. Deng et al. [94] promoted an intermediate class between the PPP and GPP, named β-GPP. The β-GPP can closely model the deployment of actual BSs with regard to coverage probability and other statistics. The use of repulsive point processes is suitable to describe the spatial distribution of scenarios with sparse requirements, such as rural areas [94]. A new Matérn-II-discrete process [85] is developed to approximate the distribution of IEEE 802.11p transmitters in VANETs. The Matérn-II-discrete process can jointly capture the temporal state of the back-off stage and the spatial distribution of the transmitting nodes. A comparison of regular point processes is provided in Table 2.

Based on the above models, the deployment of communication infrastructure, such as BSs, can be adequately designed, and the demand for electric vehicle charging stations can be reasonably estimated. Most of the models are confined to one- or two-dimensional spaces at present. There are relatively few models to capture the increasingly popular viaducts and other three-dimensional traffic network conditions. The development of three-dimensional models will be relevant to the future research of autonomous vehicles including ground and aerial.

3.4 Time-domain vehicular distribution model

The temporal distribution models typically characterize the arrival processes of vehicles to service sites, such as base stations and road site units. The models can be used to explore the time-variant requirements of vehicles to access V2I communication networks.

A Poisson arrival location model (PALM) [63] does not account for interaction among vehicles on highways. The experiment concludes [63] that although the queue lengths are independent at each time, there is a dependency between the queue lengths at different times. Then the specific implementation of PALM in a Markovian highway is proposed in [64, 65]. In the Markovian highway, the process of vehicles arriving at the entrance according to a non-uniform Poisson process in time, the state of each vehicle evolves according to a non-stationary continuous time Markov chain while the vehicle moves deterministically along the highway.

Refs. [64, 65] studied the performance of the network by calculating the call density and the handoff rate of communicating mobiles on a Markovian highway. It is shown in [64] that the introduction of inter-vehicle interaction can disrupt the PALM's Poisson characteristics. The authors of [66] extended the PALM to a one-way, semi-boundless urban road system with traffic lights. The extended PALM consists of a deterministic fluid dynamic model and a stochastic point process model. The mean flow rate of the traffic stream and the density of vehicles are obtained from the conservation equations of the fluid dynamic model, introduced in Section 4. The randomness of individual vehicles and the probability distribution are extracted from the stochastic point process model. This model is verified by empirical loop detector data from the London Center [66].

Refs. [67,68] studied the network connectivity of mobile nodes on a unidirectional highway with multiple lanes. It is assumed in [67] that vehicles' arrival at the start of highways follows a Poisson process, as described in Figure 5. Each stream of nodes on the highway is modeled as a $M/G/\infty$ queueing system,

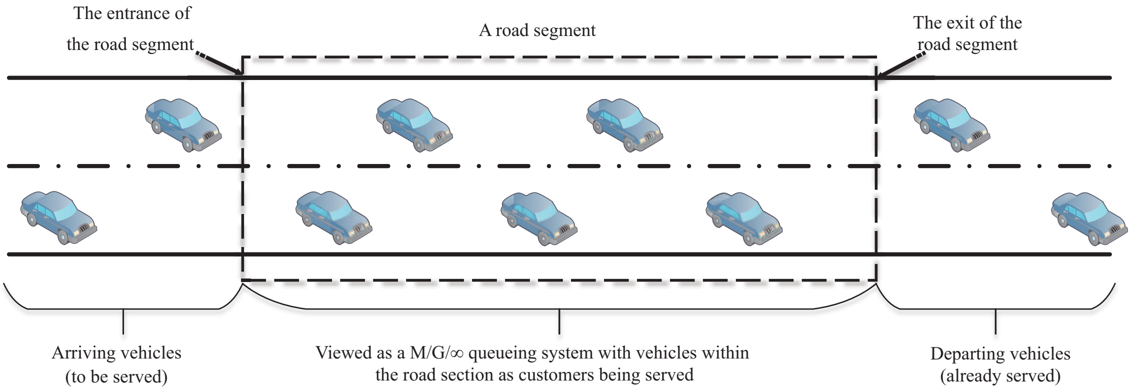


Figure 5 (Color online) A M/G/∞ highway system (see Section 3).

i.e., the vehicle arrivals are Markovian (modulated by a Poisson process), and service times have a general distribution. The unidirectional highway is divided into multiple units based on the maximum transmission range of the node. Each unit begins with a service point where the vehicles can enter or exit. For analytical simplicity, it is assumed that the number of passing vehicles is not limited by the number of lanes (or servers) on the highway and that there can be infinite lanes. For the analysis of connectivity, the inter-vehicular spacing obeys an exponential distribution. It approximates the VANETs' connectivity under dynamic traffic flows by introducing a robustness factor, which captures the effects of various disturbances, such as accidental acceleration or deceleration, and lane changing, on connectivity [68]. The factor quantifies the dynamic traffic flow characteristics.

4 Vehicular traffic flow model

Vehicular traffic flow models present another important element of vehicular mobility, in which the movement of vehicles is described mathematically in different time and space scales. This section is based on three classes of models: differential equation models, discrete Markov processes, and empirical models, as shown in Figure 2. They are compared in terms of accuracy, complexity, and practicality, as shown in Table 3 [31, 95–139].

4.1 Motivation to model traffic flow

The driving habits and mental status of drivers are variable [95]. Some researchers have summarized basic characteristics and evolutionary relationships of traffic flow from a macroscopic structure and established traffic flow models to analyze the characteristic parameters, such as flow, speed, and density [95, 140, 141]. The flow is the number of vehicles crossing a road segment of unit length in unit time. The experimental science of traffic flow is first developed in [95]. These experiments include a method for measuring the mean and standard deviations of vehicle speeds at a point or journey time, and for measuring the number of vehicles passing a given point in unit time.

The traffic performance before and after a change of road conditions was studied in [142], and statistical techniques are used to analyze whether the change can significantly reduce the journey time or avoid accidents. The mean values of the velocity were discussed in [143]. Over space means over a road segment and over time means over an interval of time at a fixed location. New statistical methods of vehicular traffic models can result in better approximations [38, 144], for example, by combining theoretical ideas with experimental data and the experience of individuals [143].

4.2 Differential equation model for continuous traffic flows

Differential equation models are based on the conservation law in physics by analogizing traffic flows to fluids or gases and analyzing the time-changing vehicular density by solving the underlying partial differential equations (PDEs) [95]. The models change deterministically over time. The differential equation models aim at recreating the formation and propagation of traffic flows and interpreting the formation mechanism of various traffic phenomena. The idea of studying traffic flows as a compressible

Table 3 The categories of vehicular traffic models (see Section 4)

Categories	Model	Characteristic	Scenario
Differential equation Model	First-order model [95–100]	Conservation equation	Simulating changes of volume, velocity, and density with time for each traffic site on highway.
	High-order model [101–110]	Conservation equation momentum equation	
	Gas-Kinetic model [111–113]	Boltzmann equation	
Discrete Markov model	Cellular automaton model [132–139]	Cellular automaton	Simulation of complex traffic systems with intersections and traffic signals for city intersection.
	Cell transmission model [114–119]	Discrete difference equation	Capture the instability of actual traffic. Simulate dynamic traffic.
Empirical model	Statistics time-series model [31, 121–124]	Linear estimators	Based on past values of modeled time series.
	Nonparametric regression model [31, 120, 121]	Pattern recognition prediction algorithms	Search for similarity information between prediction and historical data.
	Neural network model [31, 121, 125–131]	nonlinear systems	Make predictions or decisions without being explicitly programmed.

fluid is proposed in [95]. The differential equation models can be classified into first-order kinetic models, higher-order kinetic models, and gas dynamics models.

A fluid kinematic traffic flow model is introduced by Lighthill and Whitham [95] and by Richards [96] and thus called the LWR model. A flow Q is a function of the concentration k in the x space and t time, as $Q = kv$, in which v is the velocity. According to the conservation law of vehicles on a uniform highway section without exits and entrances, the LWR expression is obtained as [97, eq. 3]

$$\frac{\partial k}{\partial t} + \frac{\mathrm{d}Q\left(k\right)}{\mathrm{d}k}\frac{\partial k}{\partial x} = 0. \tag{1}$$

The LWR model [95] can reproduce the process of traffic interruption congestion and evacuation under the control of traffic lights (called shock wave).

By extending [95], the authors of [96] studied the shock wave in time and frequency of the LWR model. It is found that the influence of traffic lights on the traffic flow exhibits a thresholding effect. The influence of traffic lights is predominant for light traffic. But the disturbance surges when a critical density is exceeded, and the disturbance becomes predominant. The LWR model is based on the equilibrium speed-concentration relationship. The homogeneous steady state is usually called the equilibrium state [140]. The equilibrium traffic flow needs to meet two conditions, temporal stationarity ($\frac{\mathrm{d}v}{\mathrm{d}t} = 0$) and spatial homogeneity ($\frac{\partial \rho}{\partial x} = 0$). When one equilibrium state changes to the next equilibrium state, the speed changes suddenly. To achieve a new equilibrium speed, the acceleration should be infinite, which is inconsistent with the practical situation.

One way to address the problems that the acceleration can be infinite in the LWR model is to follow the basic principles of fluid dynamics and add a momentum conservation equation to form a 'high-order extension' [109]. The high-order kinetic model describes a phase transition mechanism from equilibrium to non-equilibrium and between the non-equilibrium states by adding the second-order characteristic and its associated waves. This section describes several representative high-order models, namely the PW model [101, 102], the Kuhne-Kerner model [103, 104], the Michalopoulos model [105], the ARZ model [106–109], and the generic second-order model family [110]. In Table 3, we summarize the characteristics of these models and compare them with the lower-order models in terms of accuracy and practicability. The problem in high-order expansion is how to establish an anisotropic non-equilibrium model.

A small subclass of the differential equation models is based on kinetic gas theory. This is a mesoscopic model between the fluid dynamics model and the cellular automaton model. As the last item of the continuous medium model (see Table 3), the characteristics of the gas dynamics model are described, and an objective evaluation of its practicability is provided. The gas-kinetic models emphasize the interaction among vehicles in traffic flow. A similar Boltzmann equation is established by introducing a particle distribution map. This theory is first applied in [145]. Because of some overly restrictive assumptions, this model gives good results only at low density and breaks down completely at high density [111]. Based on [145], many improved models have emerged, by modifying the acceleration term, introducing the velocity correlation between consecutive vehicles, investigating the interaction between adjacent lanes, and considering space requirements [111–113, 146].

Remark. Although the gas-kinetic-based models have a good theoretical foundation, the underlying equations contain too many parameters to be determined. Such complexity results in a slower development than other traffic flow models [12].

An extended LWR model was proposed in [97], which takes into account the distribution of heterogeneous drivers characterized by the choice of speeds in a traffic stream. Authors of [99, 100] proposed a stochastic partial differential equation (SPDE) model by adding a stochastic coercive function to the LWR model. The SPDE model can capture part of the stochastic nature of the traffic flow evolution and improve the accuracy of prediction.

Traffic data from different sensors is heterogeneous, which is a challenge in traffic state estimation. Refs. [97, 99, 100] described the space-time relationships of the data based on experimentally captured data from local fixed traffic sensors, such as inductance loops traffic data. This descripition [97] can capture the changes in the vehicle number over time. Lagrange sensor data (data from probe vehicles, consisting of, e.g., location, speed and travel time, and potentially time and space headways at different polling intervals) has been added to predict real-time traffic condition and distinguish different traffic phases [99, 100]. Ref. [98] revealed that the LWR model in the Lagrangian descriptions performs better than in the traditional Eulerian descriptions.

A comparison of the three differential equation models is provided in Table 3. The entrance and exit sites of highways are fixed. It is convenient to establish vehicle conservation equations on highway. The differential equations have been used to reproduce empirically observed velocity-density relationships and unstable traffic flows of highway site. The differential equation models have innate advantages of explaining the generation mechanism of various traffic phenomena. However, the characteristics of individual vehicles are not taken into account in the differential equation models. Subsection 4.3 introduces the discrete Markov models.

4.3 Discrete Markov process model for traffic flows

The discrete Markov models represent each vehicle motion as a discrete Markov process. Due to the repeatability of urban bus routes and the similarity of driving conditions, a Markov chain model was established according to the driving conditions of Chongqing 303 bus line. The Markov chain model serves as the predictive model to predict demand torque over a finite receding horizon instead of the nonlinear predictive model. The predictive results are related to traffic flows in real world [147]. The authors of [148] assumed the traffic network's dynamic process satisfies the Markov property that the future state of the traffic network is conditional on the present state. The traffic network was modeled as a graph and the transition between network-wide traffic states at consecutive time steps was defined as a Markov process. Then missing traffic states can be inferred step by step. As shown in Figure 6, the position of the vehicle at a certain time is regarded as the state of the vehicle, and the Markov model estimates the state of the vehicle at the next moment based on the state transition matrix. In a discrete Markov model, the characteristics of drivers, vehicles, and streets can be appropriately considered. In this section, cellular automata models (CAMs) [132] and cell transmission models (CTMs) [114, 115] are taken as representatives of the discrete Markov model. The CAM and CTM are compared in terms of feasibility and practicability, as shown in Table 3.

With the increase of vehicle density, the phase transition from laminar flows to start-stop-waves has attracted more interests [114]. This leads to discrete Markov models that can describe the transition from laminar flow to turbulent behaviors. The Markov model, unlike the continuous kinetic model, is discrete, and considers the movement of the vehicular traffic as a state transition. It simplifies the actual process of traffic flow transmission and offers a simple method for computer simulations. Also, the discrete Markov model can simulate the complex nonlinear features of traffic problems and provide more intuitive visualizations of platoon formation and dissipation, compared to the differential equation models, as summarized in Table 3.

4.3.1 *Cell transmission model*

The CTM [114] predicts the traffic behavior of the whole road by evaluating the flow status of the selected observation points (including the entrance and exit of roads). In the CTM [114, 115], the highway is divided into homogeneous sections (cells). A vehicle in a cell can only move forward to the next cell. This method can greatly simplify the continuous differential equation model, but does not directly address

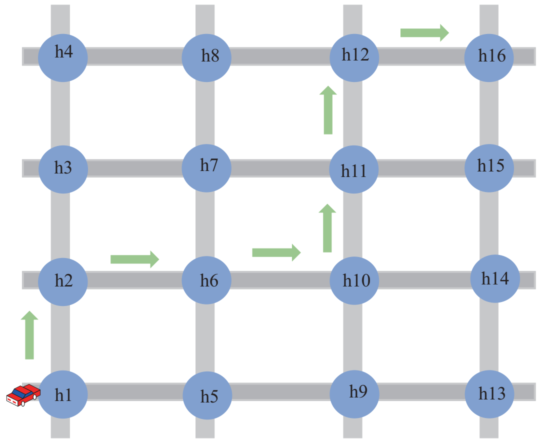


Figure 6 (Color online) An example for predicting the mobility of vehicles using the Markov model. The green arrow indicates the most likely transition trajectory of the vehicle state.

the exit flow of traffic like the kinematic model, as given by [114, Eqs. 1a and 1b]

$$\begin{aligned} &n_i\left(t+1\right)=n_i\left(t\right)+Q_i\left(t\right)-Q_{i-1}\left(t\right), \\ &y_i\left(t\right)=\min\left\{n_{i-1}\left(t\right),Q_i\left(t\right),\left(\frac{\omega}{v}\right)\left[N_i\left(t\right)-n_i\left(t\right)\right]\right\}, \end{aligned} \tag{2}$$

where the subscript 'i' denotes the i-th cell, $n_i(t)$ denotes the number of vehicles in i-th cell at time t, and $y_i(t)$ is the number of vehicles that can flow from the $(i-1)$-th cell to the i-th cell in the interval $(t, t+1)$. The two constants $N_i(t)$ and $Q_i(t)$ are the max spatial capacity of the i-th cell at time t and the max input capacity of the traffic flow from the $(i-1)$-th cell into the i-th cell in time interval $(t, t+1)$, respectively. $N_i\left(t\right)-n_i\left(t\right)$ represents the amount of empty space in i-th cell at time t, and ω is the back propagation velocity of congestion disturbance. CTM is the discrete version of the LWR model.

The derivation of (2) in [114] verified that the basic CTM is equivalent to the differential equation model in terms of traffic prediction. The superiority is that the CTM captures the instability in real traffic, which cannot be covered by the dynamics model. Data from the inter-state highway I-880 in California has been used to verify the performance of the CTM on a single homogeneous highway link. However, it has been observed in [115] from measured data that the CTM is unable to reproduce the "capacity decline" phenomenon. There exists a reduction in the capacity of a congested highway when the density decreases, such as a bottleneck or an on-ramp. As illustrated in Figure 7, when the vehicle 'A' enters the highway, the vehicle 'B' is expected to perform emergency braking for avoiding 'A'. The chain reaction is that the vehicle 'C' performs the subsequent braking. In comparison to lane 2, the density and the flow of lane 1 decline.

The lagged cell transmission model (LCTM) was proposed in [149], which allows variable cell lengths and a nonconcave flow-density relation. The cost is an additional storage space, which is used to store the traffic density of the downstream in the past R time intervals. The LCTM is suitable for modeling the intersections and inhomogeneous highways, by fetching time-lagged downstream traffic density. Daganzo notes that a period of a minute or two (spanning between 1 and 2 highway miles) where rather dense traffic appeared to be coasting toward the end of the queue (like lane 2 in Figure 7). It can be used to explain the coasting effect or to help refute the existence of this phenomenon [149].

The enhanced LCTM (E-LCTM) was proposed in [116]. It adds two terms to the CTM (the first one is the sending function without considering the lag to ensure the demand no greater than the available; the second one is the receiving function without considering the lag to ensure the supply no greater than the available storage capacity). The E-LCTM discretizes the LWR model in both time and space. Many important traffic phenomena, such as queue build-up and dissipation, and backward propagation of congestion waves, are captured [116].

The switching-mode model (SMM) [117] is a linear time-varying model. It uses density (instead of occupancy) as its state variable. There are five state transition modes in the SMM model, which are described by linear equations. According to the congestion state of each highway section, different equations are chosen. However, the SMM assumes that there is at most one state transition in the highway section [118]. The authors of [118] developed a stochastic cell transmission model (SCTM) to

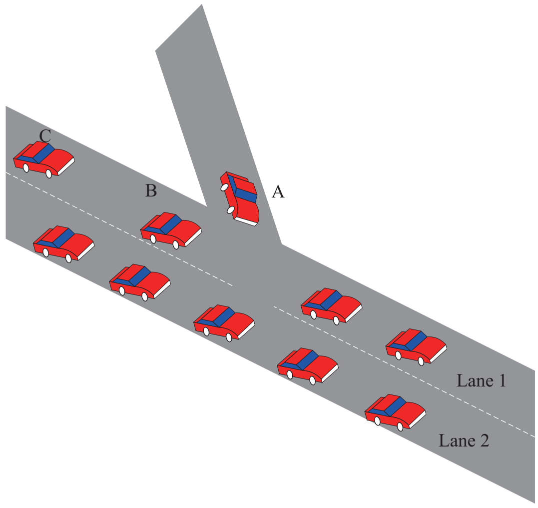


Figure 7 (Color online) An example of 'capacity decline' at an on-ramp. When the vehicle 'A' enters the highway, the vehicle 'B' is expected to perform emergency braking to avoid 'A'. The chain reaction is that the vehicle 'C' performs the same braking. In comparison to lane 2, the density and the flow of lane 1 decline.

capture the uncertainties of demand and supply. A highway is decomposed into several short segments, and each segment is represented as a sub-system consisting of two cells that satisfy the SMM hypothesis that the road segment has at most one wavefront. The SCTM has an advantage in both computational time and computer memory. Empirical traffic data [118] measured by PeMS (performance measurement system) is used to validate SCTM through two scenarios of uncertain supply and demand.

An asymmetric cell transmission model (ACTM) was proposed in [119] to solve the on-ramp metering control problem. The model is numerically tested using the data of a heavily congested highway in Southern California. The simulation results show that the delay can be reduced by 17.3%. A traffic light extension to the CTM was presented in [150], and the traffic congestion, as well as its spread over multiple intersections, can be reproduced. A distributed traffic control strategy based on the CTM was presented recently [151]. A series of rules for updating the intersection status are designed by the sub-gradient descent method. Under the control of the signal light, all this compensates for the lack of the CTM in describing the evolution of traffic flow at the intersection [151].

Remark. The CTM is actually an approximation to the LWR model, which is a currently recognized technical method to model traffic flow and simulate dynamic traffic [114–119, 150, 151].

4.3.2 *Cellular automaton model*

The one-lane CAM is defined on an array of lattices with open or periodic boundary conditions in the Nagel-Schreckenberg model (N-S model [132]) and Fukui-Ishibashi model (F-I model [133]). The status of each lattice is busy (occupied by a vehicle, expressed by 1) or empty (expressed by 0). State transitions involve acceleration, deceleration, distribution of position, and vehicle movement. The difference between the N-S model and the F-I model is that the velocity of the N-S model is limited, while the other model has no limit. The N-S model [132] can reproduce some real-traffic phenomena, such as a realistic flow-density relation.

The one-lane CAM can naturally transition from a laminar flow to a turbulent flow in a Monte-Carlo simulation with a computational advantage, as compared to CTM [132]. In the numerical simulation, a phase transition between a jamming phase and a flow phase occurs as the vehicle density increases [152]. Moreover, this transition occurs at or near the point of the maximum throughput of the traffic, which is consistent with a kinematic-based conclusion. The lifetime of traffic jams is a power law distribution, which makes the prediction of traffic flow characteristics more difficult [152]. Nagel et al. [153] developed a phenomenological theory. It predicts the critical exponents for this transition between laminar and jammed regimes.

Based on the above studies, researchers have expanded and improved the CAM by adapting it to more realistic circumstances. The limitation of the single-lane model is that it is impossible to describe the phenomenon of vehicular lane-changing and overtaking. The modern improvement direction is to modify the CA's lane-changing rules and constraints to accommodate more realistic scenarios.

The research of [134] extended the simple N-S model to a two-lane model. The effect of different lane changing rules on traffic flow is studied using the stochastic two-lane cellular automata model (STCAM) [134]. Authors of [135] extended the CAM to a two-lane model using two parallel single-lane models with periodic boundary conditions. A new asymmetric lane changing model (ALCM) was proposed in [136]. The ALCM is simulated according to the lane-changing rules of Germany on the highway, and satisfying results both in constraint of gaps and velocities are obtained. The basic symmetric two-lane CA model is updated in [137]. A lane-changing rule is added to capture the aggressive lane-changing behaviors and different lane-changing habits of different drivers. Based on simulations, the flow rate of the mixed traffic system is improved at the medium range of vehicle density, as compared with the primary symmetric two-lane CA model [137].

Some other complex urban traffic scenes, including intersections and two-way traffic, are also the potential areas for improving CAM. A two-dimensional traffic flow CA model for intersection scenes was proposed in [139]. The authors of [154] improved the velocity update rule by adding a slow-to-start rule to the N-S model. A CA model for two-way traffic interaction [138] was proposed. It describes the traffic flow on a narrow road with no road separator between the two directions along the road. The sources of the verification data are diverse, such as actual observed traffic data and experimental data obtained by the simulation platform. Ref. [155] verified the importance of calibration and validation to get a more realistic model.

Remark. Compared with differential equation methods, the CA model can be easily implemented and numerically studied. The CAM is always used in the simulation of complex traffic systems with intersections and traffic signals.

The comparisons of the different discrete models are summarized in Table 3. Since the traffic units are discrete, the discrete-continuous-discrete approximation process is avoided in the CA model, as compared to the CTM. Due to the excessive discretization of space and time, there may be some additional problems, such as the tradeoff between the grid and time-step sizes, the disagreement between discretization and real dimensions, diagonal motion trajectories, and limited speed range and maximum density.

4.4 Empirical model for traffic flow

The last category of vehicular traffic models includes empirical models, which are developed based on measurement data. Traffic flow prediction models are based on observational data (historical data and real-time measurement data) [31, 121]. Such models are generally more effective, as it is very difficult to drive knowledge-driven simulation models (e.g., due to the lack of understanding of the underlying physical processes), or the available models are not adequate [34]. Characteristics of these models are shown in Table 3.

The mathematical models of traffic flow with nonlinear equations may suggest a nonlinear kinematic system, which is difficult to analyze in general. Continuous dynamic models are computationally challenging with limited flexibility. The drawbacks of excessive discretization often accompany the discretized Markov model. In order to explore the traffic pattern of a region, there are many data-driven models based on historical and real-time information. A description of the data sets used by existing studies is listed in Table 4 [125, 126, 128–131, 156–165], which also creates a new avenue to analyze big vehicular data by machine learning.

4.4.1 *Autoregressive integrated moving average model*

Autoregressive integrated moving average (ARIMA) models rely on an uninterrupted series of data. Such a time series model is mathematically well known, but may be unfit for traffic forecasts of a wide region [31, 121], or incapable of handling missing values. The treatment of ARIMA can be found in [50]. Williams et al. [122, 123] proposed a seasonal ARIMA model to predict the traffic flow. Compared with the nearest-neighbor, the neural network, and historical average models, the seasonal ARIMA performs better in the single-interval traffic flow prediction [123]. The experimental data are taken from the Virginia Department of Transportation's Northern Virginia traffic management system. A seasonal ARIMA model with limited input data was presented by Kumar and Vanajakshi [156]. Only the previous three days of flow observations are used as the input to predict the flow values of the next day (24 h ahead of forecast). The traffic flow on a three-lane arterial roadway in Chennai, India, was predicted [156].

Table 4 Data description of references

When	Where	Model	Description	Ref.
From May 26 to June 6, 2014 (excluding weekend)	The automated traffic sensor installed at Perungudi, sensor, the Collect-R camera, is permanently fixed, and is far away from the study location about 3 km.	ARIMA	The raw traffic flow data contains each 1 min class-wise traffic flow for one day.	[156]
From October 1, 2012 to November 19, 2012	Come from the deployed loops NS-NO24, (1), (2) in North-South Elevated Road of Shanghai.	KNN	The time interval of the collected traffic flow data is 20 s.	[157]
From May 1, 2013 to June 1, 2013	G6 Beijing-Lhasa Expressway in China	KNN	Through a mature micro traffic simulation model, the traffic flow of all road segments are calculated from the highway traffic toll data.	[158]
A total of 24 data sets with each data set covering a single day are used.	In a 7-mile long freeway segment on Interstate 880, the Nimitz Freeway, in Alameda County, California, between Alverado Niles Road and S/R 238 for the Freeway Service Patrol (FSP) program	Gauss MLE	Data for each day covers a time period between five to ten in the morning and two to eight in the afternoon.	[159]
16 days	The Performance Measurement System of California freeway	SVM	Five-min traffic flow data from 5:00 am to 10:00 am, including loop-detector data and traffic incident data.	[125]
From September 1, 2007 to October 31, 2007	A three-lane freeway in the 3rd ring freeway of Beijing	FNN	Each record includes 5-min traffic flow volume, 5-min traffic flow average speed, and 5-min traffic flow average occupancy.	[128]
From January 1, 2005 to December 30, 2005	A detector on National Highway 107, Xia Yuan, Huangpu, Guangzhou, Guangdong, China	ARIMA FNN	The traffic flow data is aggregated and averaged into 1-h periods, 24 h/day.	[126]
Twenty-one-week period from the first week of January to the last week of May in 2009	Automatically collected by the toll collection system and managed by Center for Operations Analysis and Supportive Information System, an ADMS at Korea Expressway Corporation	KNN NPR	There are a total of 13440 data, each record contains date, sequence number, and traffic volume.	[129]
At 18:00, and 1:00.	Road I-880S in Alameda County, Bay Area, California	Lognormal model	The sampling period of the flow and speed data ranges from 30 s to 5 min.	[130]
From January 1, 2013 to March 31, 2013	PeMS	SAE	The traffic data are collected every 30 s.	[131]
From March 1, 2012 to June 30, 2012	Loop detectors in the highway of Los Angeles County	RNN	Select 207 sensor	[160]
From January 1, 2017 to May 31, 2017	PeMS		Select 325 sensors in the Bay Area.	
From February 1, 2017 to March 26, 2017	Guangzhou, China	LSTM CNN	These features include temporal features, spatial features, meteorological features, and event features.	[161]
—	NGSIM I80-1	CFM	The data sampling period is chosen as 1 s.	[162]
From 7:50 am to 8:35 am on June 15, 2005	The southbound direction of US Highway101 in Los Angeles, CA, USA	DNN	1535 vehicle pairs and 944974 seconds of vehicle trajectories	[163]
From 4:00 pm to 4:15 pm	In the San Francisco Bay Area	LSTM	Contains 15 minutes of vehicle trajectory data collected using synchronized digital video cameras providing the vehicle lane positions and velocities over time at 10 Hz.	[164]
From 7:50 am to 8:35 am, on June 15, 2005; from 4:00 pm to 4:15 pm and from 5:00 pm to 5:30 pm, on April 13, 2005	A segment of southbound US Highway 101 in Los Angeles, CA, USA	Rough set theory	Both data sets represent two traffic states: conditions when congestion is building up (period of the first 15 min), which are denoted as the transition period, and congested conditions (period of the remaining 30 min).	[165]

4.4.2 *Nonparametric regression model*

Nonparametric regression models do not have explicit (closed-form) mathematical expressions. The models search a historical database for the similarities of the prediction library and estimate the predicted values based on pattern recognition and prediction algorithms. They are able to handle the prediction of abnormal traffic conditions. The three steps of the nonparametric regression model are based on a historical database, the search of the nearest neighbors, and implementing the prediction. Different characteristic parameters may be stored in the historical database, which generally includes the characteristic parameters of the neighbor node and the node to be predicted. The difficulty of the nonparametric regression model [31, 121] is to identify "neighbors".

In [120], the nonparametric model is improved in terms of traffic flow forecast by coupling it with heuristic forecast generation methods. Although the prediction accuracy of the nonparametric regression model does not reach the level of the seasonal ARIMA, this provides a new idea for improving the performance of the nonparametric regression models [120]. The K-nearest neighbor nonparametric regression model (KNN-NPR) [157, 158] is a representative of the NPR models. The idea is a simple single-interval prediction and a more flexible multi-interval prediction. The flexible multi-interval prediction can find matching time series of different lengths for various adaptation scenarios and improve the prediction accuracy [157, 158]. The description of training data is shown in Table 4.

The linear nature of these time-series models is unable to capture the dynamics and nonlinearities present in traffic flows, limiting their capability to predict long-term traffic flows accurately.

4.4.3 *Support vector machine model*

A regression online support vector machine (OL-SVM) [125], based on supervised statistical learning, can predict the short-term traffic flow of highways under typical and atypical scenarios. The typical traffic scenario provides the traffic patterns without any unexpected incidents, such as accidents or crashes. In the atypical traffic scenario, the testing samples are from either special days (such as public holidays and days with special events) or correspond to an emergency occurring (such as traffic accidents). The OL-SVR is shown to outperform the GML in the atypical scenario [125]. In short-term traffic flow prediction, SVM is utilized to realize the closed-loop optimization in the selection process of related links [166].

4.4.4 *Neural network model*

The author of [159] proposed a prediction model based on the Gaussian maximum likelihood estimation (GMLE) method. The GMLE model uses 5 minutes of traffic flow data for one-step prediction. The data is a combination of real-time and historical traffic information collected from a section of 7-mile highway in the California PATH Database which is described in Table 4. Compared with the historical average model (HAM), non-parametric neighborhood model (NNM), and linear regression model (LRM), the GMLE model is superior in terms of absolute deviation and mean square error [159]. Bayesian neural networks are introduced to predict traffic flow in the short term, based on conditional probability theory and Bayes rule [128].

A dynamic wavelet neural network model was proposed for traffic flow forecast recently [127]. The model incorporates the self-similar, singular, and fractal properties discovered in the traffic flow, to achieve high accuracy for the forecast in both the short term and long term. A Bayesian regularized artificial neural network (BRANN) is designed to predict vehicle trajectories. It is shown that the proposed approach can timely evaluate dangerous events and realize safe driving in terms of collision avoidance and lane-keeping [167]. The authors of [128] introduced an adaptive prediction method based on a Bayesian combined neural network (BCNN). The BCNN model combines multiple single neural network predictors by assigning different credit values to each predictor. RNN is commonly used to capture the time dependence in traffic prediction. The authors of [168] used its variant long short-term memory (LSTM) to predict traffic Flow. Some researchers propose a Flow Conv GRU model to predict traffic flow or traffic state in [169–171]. An extended causal convolutional neural network (DCCNN) was proposed to predict short-term traffic flow [166].

Neural networks (NN) can handle nonlinear problems, due to their nonlinear activation functions and multi-layer superposition (e.g., ARIMA can only capture linear relationships 4.4.2). The authors of [126] aggregated them to produce a better result. Using the measured data to train the neural network, the hourly, daily, and weekly intervals are exploited to describe the periodicity of the data. In [129], a methodology that combines deep learning techniques with the KNN-NPR model was proposed. Compared with stacked auto-encoder (SAE), BP-NNet, and OL-SVM, the hybrid model is suitable for congested traffic in an urban area, and the prediction accuracy is improved. Ref. [172] proposed an integrated model based on dynamic Bayesian network (DBN) and LSTM that combines the intention recognition and trajectory prediction of vehicles in an unsignalized intersection scene. The DBN is used to infer the distribution of intentions at intersections to improve the prediction time. The LSTM with encoder-decoder is used to predict trajectories to improve the accuracy of prediction. Further deep learning possibly developing an even nicer approach remains open in research. The matrix decomposition method is introduced into the deep learning framework to improve global prediction ability [173]. Ref. [174] proposed a novel method for long-term speed prediction that aims to build a mapping model between the driver-vehicle-road-traffic characteristic parameters and vehicle speed. The proposed GA-BP algorithms can enhance the accuracy and robustness of speed predictions for different road types.

4.4.5 *Traffic flow prediction model with time and spatial correlation*

Most of these models utilize the freeway data to establish a single-variable traffic flow predictor about a fixed observation point [50]. The above models do not capture the spatial dependence between traffic time series, and subsequent researchers extend the method of processing multivariate time series. Stathopoulos and Karlaftis [124] established a multivariate state space approach to the time-series model by using urban arterial street information near the center of Athens, Greece. The multivariate state space model is superior to the ARIMA model in terms of the average prediction errors. The short-term prediction of traffic flows in signalized urban arteries is still not as accurate as on freeways.

To understand the dynamics of the network topology of VANETs, the authors of [130] integrated real-world road topology and real-time data extracted from the PeMS into a microscopic mobility model (SUMO) to generate realistic traffic flows along a highway. Ref. [175] used a kinematic origin-destination (OD) estimation matrix to generate traffic flow data at all links based on demand, historical data, and the limited real-time data by using an online optimization methodology. An autoregressive model [175] is trained which can adapt itself to unpredictable events.

Deep learning algorithms can express traffic characteristics without prior knowledge and have excellent potential in traffic flow prediction [176]. A deep learning traffic flow prediction method was proposed in [131], where the inherent temporal and spatial correlations of traffic flows are captured. A deep learning-based offline algorithm was proposed to predict vehicle mobility in a future time period, aiming to collect maximum sensing data, and the input feature of the algorithm is historical vehicle trajectory. Then a greedy online algorithm was proposed to recruit a subset of vehicles with a limited budget. The proposed model is evaluated on a real taxi data set, and experimental results show that the proposed model achieves better performances [177]. The SAE model [131] is used to learn general traffic flow features, and it is trained in a layer-wise greedy fashion. The method is proposed to model traffic networks as images [160, 161, 178, 179] to capture the spatial dependence of traffic. Convolutional neural networks (CNNs) and graph convolutional neural networks (GCNs) are used to extract spatial features. Traditional CNNs are effective in dealing with grid-like data which can be represented by a one- or two-dimensional matrix. However, traditional CNNs ignore the topology of the underlying transportation network, and the performance is greatly reduced in practical applications [179]. A model based on graph convolution network (GCN) is applied to predict short-term traffic flow of large-scale urban road network [180]. Road traffic conditions are influenced by other road traffic conditions and are highly time-dependent. To simulate this property, spatial attention mechanisms are usually used to capture the correlation between regions in the road network [171, 181, 182]. GCNs [179] apply an attention mechanism to graph convolution and have the great potential to deal with irregular data that does not have a regular spatial structure, for example, data where the connection numbers of nodes can be different.

5 Drivers behavior model and fleet pattern

Another important aspect of vehicular mobility is driver behaviors, where the greedy driving strategies under various traffic conditions are developed. Specific driving strategies to reach the destination include route planning, car following, lane changing, velocity controlling, and direction controlling. The direction- and velocity-controlling strategies rely on various sensors to collect information and use the engine and torque control (for instance [183, 184]). These two models of driving strategy considering mechanic and control mechanisms are beyond the focus of this article.

Car-following models and lane-changing models can be established by analyzing real-time information from vehicular communication networks and transportation networks. The general scheme for driver behavior models is shown in Figure 8. The controlled vehicle adjusts itself based on states of surrounding vehicles. Extensive and detailed vehicle trajectory data help to develop driver behavior models [15]. This section introduces the lane-level vehicular movement model from three aspects, car-following models, lane-changing models, and vehicular trajectory models. Table 5 [162–165, 185–197] summaries some applications for communication and transportation, which is also vital to autonomous driving technology.

5.1 Motivation to model driver behavior and fleet pattern

An accurate prediction of vehicular motion is useful for communication resource scheduling and driving strategy. For example, the vehicle mobility management of a network is to explore reasonable access

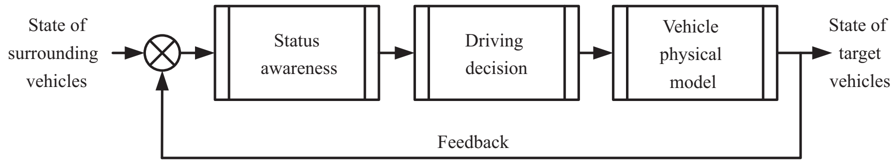


Figure 8 General scheme for driver behavior models (see Section 5).

Table 5 The comparison of driver behavior models (see Section 5)

Category		Existing work	Application	
			Communication	Transportation
Car-following model	Traditional models	[185–187]	Communication strategy	Resource scheduling
	Empirical models	[162–164, 188, 189]		
Lane-changing model	Traditional model	[165, 190]	–	Autonomous driving/advanced driver-assistance system
	Empirical models	[191]		
Trajectory prediction model	Traditional model	[192, 193]	Build VANETs, allocate network resources	Dynamic traffic control
	Empirical models	[194–197]		

schemes based on real-time mobility prediction. Another example is platooning which is a cooperative driving pattern for a group of vehicles where one vehicle follows another and keeps a consistent distance to the vehicles ahead [198].

Road and radio resource scheduling can be inspired by vehicular platoons. There is a linear relationship between spacing and speed in the car following behavior. Ref. [199] found the optimal density of the following vehicle and the maximum traffic capacity of roads by studying car-following models. The car-following models help resolve inter-vehicle communication problems. For example, inter-fleet communication can be studied by controlling the movement of the platoon leader [200].

Lane changing is another driving behavior related to reliability and autonomous vehicular technology. The safety and throughput of traffic are heavily affected by lane-changing behaviors. The linear relationship of spacing and speed would be absent in the presence of frequent lane-changing maneuvers [201]. Therefore, studying the vehicle lane-changing model and designing reasonable lane changing trigger mechanisms are helpful to improve traffic and driving safety.

5.2 Trajectory prediction model

The short-term trajectory of vehicles can be predicted in three ways.

(1) According to the fixed schedules (for example, buses), the trajectory under normal circumstances can be attained.

(2) For vehicles using navigation systems, drivers usually move following the suggested path from the navigation system, which can be regarded as the future trajectory.

(3) Various methods for trajectory prediction have been developed [17] based on data mining and theoretical analysis. This is the focus of this subsection.

5.2.1 *Kinetic method for trajectory prediction*

The trajectory of a vehicle can be modeled with geometric features since it is constrained to the roads designed with specific geometric models [202], such as the Ackermann model and the bicycle model. The kinetic equation is established according to the input information, the acceleration, speed, yaw angle, steering wheel rate, brake, acceleration pedal pressure, and so on. For example, Ref. [203] used the bicycle kinematic model to compute the future trajectories of vehicles for collision risk estimation. In [202], the authors established a car kinematic model based on the Ackermann steering geometry. Furthermore, the autonomous parking path is planned.

For the dynamically changeable vehicle states, inertia can only show good performance in the short term. Ref. [193] presented a trajectory prediction method. It combines the constant yaw rate and acceleration motion model with a maneuver recognition module. The combination rule improves the accuracy of both short-term and long-term predictions. The maneuver recognition algorithm and trajectory prediction method are tested using pre-recorded human driving data under semi-urban conditions on the 3rd and 4th ring roads of Beijing, China.

Another research direction is to use access information, such as base stations, to determine vehicle location information, and then use various signal processing methods to estimate the future vehicle location and trajectory in a short time. Ref. [192] used a robust extended Kalman filter (REKF) from the user's location, heading, and altitude to estimate the next moment of the vehicles. A method to effectively approximate the collision probability was presented in [204]. The model has good predictive accuracy for vehicles with a monotonic movement. However, Refs. [192, 204] ignored the dependencies between vehicles in the scene.

In [194, 205], the accuracy of vehicular trajectory prediction based on the Markov process was lower than that of pedestrian trajectory prediction. Increasing the order of the Markov process improves the accuracy, but the computational complexity grows exponentially.

5.2.2 *Data-driven method for trajectory prediction*

Another important technology of vehicular trajectory prediction is machine learning. The accuracy of the random waypoint models can be improved by introducing measured data of street maps, such as [206, 207]. Ref. [206] presented a realistic model of node motion based on the movement of vehicles on real street maps. It can describe the action of vehicles on roads. However, the waiting time around the intersection is not considered. A mobility model for vehicle-borne terminals in urban environments was proposed in [207]. The model accounts for the arbitrary urban street patterns and the realistic terminal movements through a limited number of parameters, such as the locations and speeds of vehicles. These parameters can be easily measured or derived from the street map.

A kernel variable-length Markov model (KVLMM) combines the sequence analysis with the Markov statistical model [195]. The KVLMM uses kernel smoothing to train with fewer data samples and thus executes training in linear time for large data samples. It reduces the overhead of data processing without compromising the prediction accuracy. A deep learning model, GAS-LED (global attention and state sharing based LSTM encoder-decoder), is proposed to learn the spatio-temporal dynamics of vehicles from their historical trajectory data sets and predict their trajectory in the near future [9]. A diffusion kernel model was proposed in [196] to describe the diffusion behavior of a vehicle in network and predict vehicles' trajectory. In a 2014 Kaggle competition, the best performing model for vehicle destination prediction is based on neural networks [197]. They use an almost fully automated bidirectional recurrent neural network (BRNN) to predict the destination of a taxi based on the start of the taxi trajectory and associated metadata. The accuracy of the aforementioned prediction models depends on the statistical conformity between the training data set and actually captured data [196].

5.3 Car-following model

Car-following is the basic driver behavior. This subsection divides the car-following model into the improved conventional models that consider driver psychological factors and new developments based on data-driven models. The applications of car-following models are summarized in Table 5.

5.3.1 *Conventional differential equation model*

Classical car-following models capture the vehicle dynamics but ignore or oversimplify the driver's psychological impact [208]. However, mental conditions such as driver distraction and delay in response can affect the driver's response time, and change the judgment of the safety distance in a model. The car-following models (CFM) can be modeled as (3), where the acceleration $\dot{v}_j(t)$ of vehicle j, is expressed as a function of the velocity $v_j(t)$, the inter-vehicle spacing to the preceding vehicle $S_j(t)$, and the velocity difference $\Delta v_j(t)$ [198, Eq. 3].

$$\frac{\mathrm{d}v_j\,(t)}{\mathrm{d}t} = \dot{v}_j(t) = f\left(S_j(t), v_j(t), \Delta v_j(t)\right). \tag{3}$$

Drivers' behavior can have a strong impact on the operations of the vehicle [208]. The performance of the models varies greatly for different types of vehicles and drivers. The potential car-following models are intelligent driver model (IDM) and full velocity difference model (FVDM). Ref. [185] proposed a heterogeneous car-following model which consists of low- and high-sensitivity vehicles based on the FVDM. The vehicles can have different reaction sensitivities while the drivers have the same reaction times. Lindorfer et al. [186] improved the IDM by adding situation-dependent reaction times, different types of driver distraction, and driving errors. It is called the enhanced human driver model (EHDM).

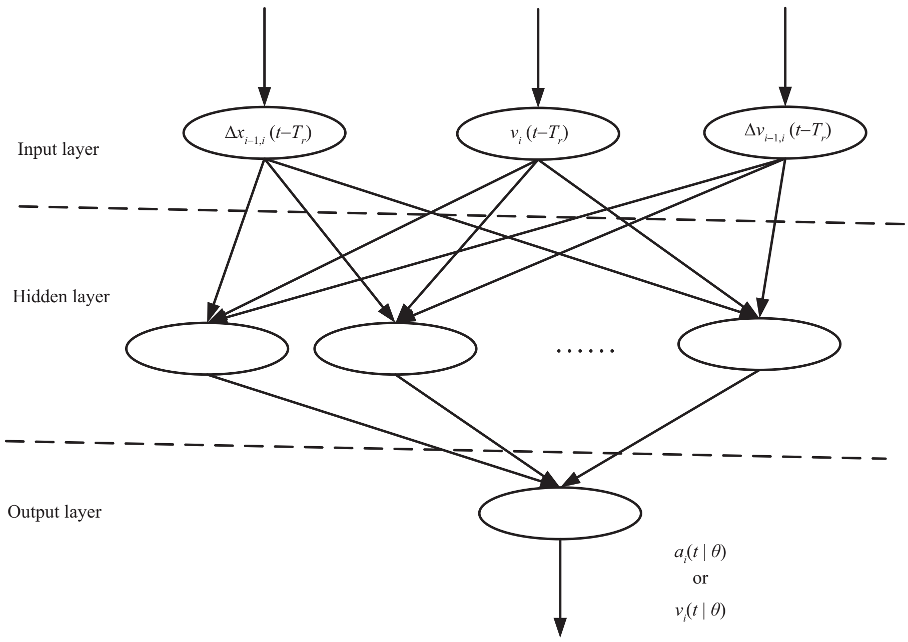


Figure 9 The structure of feed-forward neural networks for CFM (see Section 5).

The EHDM considers the different effects of minor or severe distractions on the reaction time, using a stochastic Wiener process to model the error behavior of driving.

The modal car-following model (MCFM) [187] adds three human factors (namely, estimation error, response delay, and time expectation) to the CFM. The three factors change along with a random model. It is possible to capture the physical dynamics of a vehicle, as well as the different drive modes of a driver [187]. The MCFM is tested using the Mathworks Simulink tool, with varying degrees of improvement compared to the IDM and FVDM.

The aforementioned CFMs capture the influence of drivers and other factors on the car-following behavior by introducing specific parameters in the mathematical equation. There are two further challenges.

(1) The computational complexity increases as the number of parameters increases [209].

(2) Researchers typically calibrate the parameters of the CFMs based on the traffic data in a particular scenario. Such models are thus specific rather than universal.

A sensitivity analysis method [209] is proposed to evaluate the influence of the parameters on the model, and the importance of every parameter of the model is ranked.

5.3.2 *Empirical model*

A new way to reduce human interference is to develop data-driven CFMs via machine learning or artificial intelligence. The empirical CFM can describe various traffic characteristics. Some studies in recent years are as follows. Figure 9 shows the structure of the feed-forward neural network CFM [163].

The fuzzy logic-based model was proposed in [162,210]. The fuzzy logic-based model captures a number of real factors, such as the count of vehicles ahead of the followers in their lanes, and the type of the leader and followers. In [162], a CFM based on the rough set theory [211] was introduced. The rough set theory extracts the car-following decision rules from an experimentally measured data set, and the follower's behavior changes according to the matching rules. The challenge is to define fuzzy sets and the associated membership functions [162].

Researchers further apply the feed forward neural network and the fuzzy neural network to describe the different behaviors of drivers. For example, the product adaptive neural fuzzy inference system (ANFIS) was improved in [188] by combining an artificial neural network (ANN) with the fuzzy inference system (FIS). The simulation results verify that the improved ANFIS model is superior to other ANFIS in forecast performance. The authors of [189] combined an autoregressive acceleration dynamic movement mechanism with a cautious car-following model to describe the traffic flow.

A shallow neural network is likely unable to capture complex driving behaviors. Deep learning is used for CFM to learn the driver behavior from actually observed empirical tracking data [163]. The model takes the velocity, the velocity difference, and the position difference observed in the past 10 s as the input and directly outputs the estimated value of the next NN layer. The deep learning model fully considers the driver's memory effect and predictive ability. Morton et al. [164] developed a neural CFM based on the measured driving data. A simulation model is used to verify the effectiveness of the LSTM recurrent neural network in predicting the acceleration distribution of vehicles on the highway.

5.4 Lane-changing model

The lane-changing models (LCM) have been widely used through computer simulations. The LCMs include mathematical (closed-form) models (such as the cell automation model [136] and the Markov-based model [212]) and data-driven models (e.g., artificial intelligence models). Only the states of host vehicles and adjacent affecting vehicles are captured in the mathematical models, and the driver's historical experience or predictive capabilities are typically not incorporated [15]. This unfortunately compromises the flexibility and accuracy of the models.

Data-driven models are effective to account for the differences between drivers, as well as the differences in the behavior of the same driver [15]. The reasons are as follows.

(1) The training of the model uses real data. And the decisions for lane-changing behavior are based on a comprehensive judgment of the vehicle-driver state.

(2) It considers the human's imprecise perception. The model parameters are verified using the optimization algorithm.

Some progresses have been made in the past decade to develop the LCMs using data analytics. A hybrid classifier is used in [165], which combines the Bayesian classifiers with the decision tree methods. The detailed vehicle trajectory data are taken from the next generation simulation (NGSIM) data set. The hybrid classifier is developed and tested using US Highway 101 and Interstate 80, respectively. The combined classifier is superior to the separate classifier in the accuracy of atypical events. The authors of [190] applied the social force (SF) behavior theory and the data-driven models to the operational level of modeling lane-changing behavior. They use the US Highway 101's empirical trajectory data (speed, position, speed difference, and position difference with surrounding vehicles) extracted from the NGSIM data set to train and test the deep neural networks (DNN-LC) and the gated recurrent unit neural networks (GRU-LC). Ref. [9] proposed a strategy to guide AV lane change. Based on the results of an improved LSTM model to predict the movement of surrounding vehicles, a guidance strategy of AV lane change and speed change is proposed, aiming to maximize the average speed of AV and minimize the impact of AV on surrounding vehicles.

Remark. Those above driving behavior models (including CFM and LCM) simplify or ignore the effects of the road geometry (such as horizontal, vertical, sag, and crest curves) and environmental factors (such as road surface and lighting conditions).

The traditional mechanical model has a clear physical interpretation and serves as the basis for studying driver behaviors. The foundation of the data-driven model lies in data, with the following challenges to establishing an effective driver behavior model.

(1) Measurement errors resulting from hardware defects may cause the data fragmentary, inaccurate, or inconsistent.

(2) Data sets may include superfluous or invalid data records. Hence there is a need to clean up the data first.

(3) Different weights should be set for various attributes that influence the behavior, for example, the roadway geometry.

(4) Some attributes that affect the behavior of the vehicles are difficult to quantify [15].

The data-driven models have gained preference in the field of lane-level modeling. The models can describe the uncertainty and characteristics of drivers. The method combining traditional mathematical equations with machine learning methods has a promising future.

6 Road network model

Transport network models use data structures to store spatial and semantic information of roads. Road networks can have a strong impact on mobility and network characteristics of wireless networks, including wireless ad hoc networks [41, 51, 213]. Establishing a large-scale road model that describes the road topology can provide necessary information for network deployment. In the context of network and

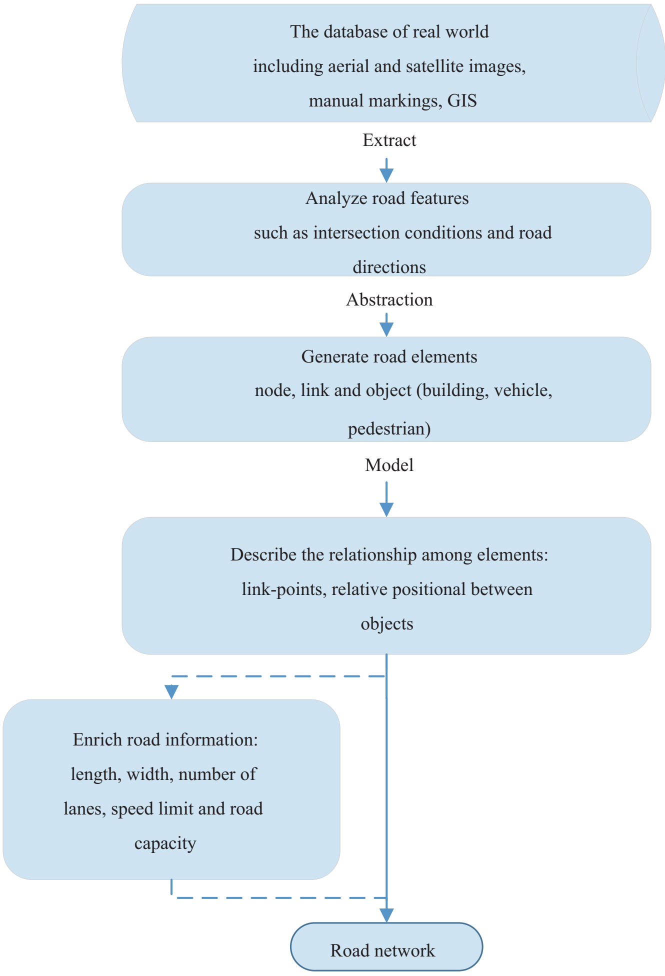


Figure 10 (Color online) The block diagram of a road modeling process (see Section 6).

Table 6 The comparison of road models (see Section 6)

Model	Example	Features of road network	Ref.	Measurement
Road network graph	Figure 12	Topology of road networks	[44, 214–218]	Usability, storage efficiency
Stochastic geometry	[80, Figure 1]	Roads	[80, 219, 220]	Usability
Rectangle tree	Figure 13	Objects on road	[48]	Storage efficiency
Mathematical cure	$L_{m(s)}$	Length/number of street segments	[221–223]	Accuracy
Phase matrix	Figure 12	Number of lanes	[47, 224–226]	Accuracy

vehicle mobility management, the movement of vehicles is subject to road shape and topology (such as lanes and traffic lights). On the other hand, a small-scale road model can provide road features for studying reasonable network handover, access schemes, and unmanned aerial vehicle (UAV) deployment. Therefore, road models are an important step to study the communication of ITS [41, 51]. This section categorizes existing transport network models based on road elements, as summarized in Table 6 [44, 47, 48, 80, 214–226].

The road network graphs and stochastic geometry models are typically used for large-scale models. The phase matrix and rectangle-tree (R-tree) [48] are chosen for small-scale modeling. We describe the four models in terms of the road elements, application scenario, and the characteristics of models. The flowchart of road modeling processes is shown in Figure 10.

6.1 Large-scale road model

Topology plays a central role in road networks. High-level descriptions of a road network, as a whole, particularly focus on network topology, road density, and other statistic aspects of roads, rather than

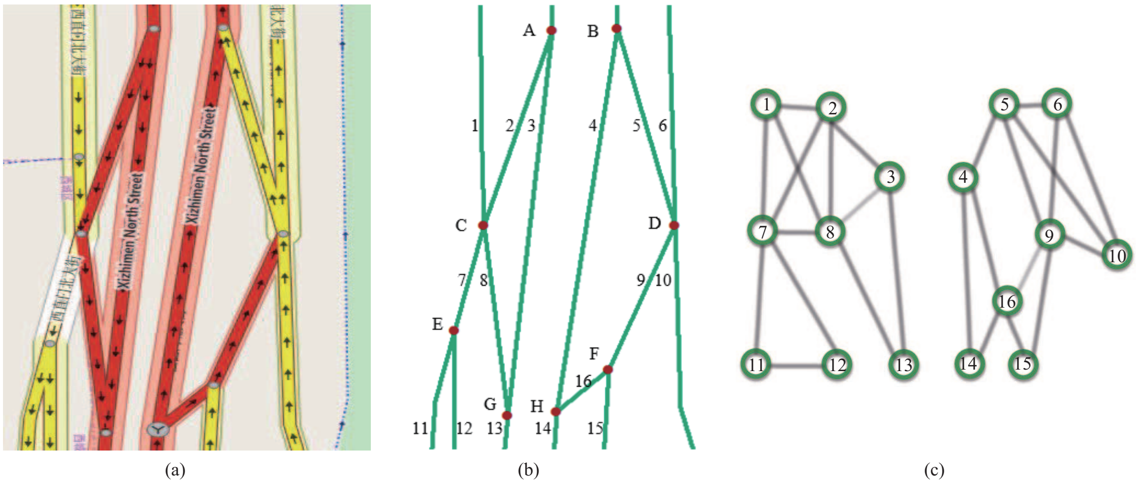


Figure 11 (Color online) Approaches to model street graphs. (a) The base map used to create street network models; (b) the junction-based street network graph, in which the red vertices represent junctions and the green edges are roads; (c) the street-segment graph, in which the green vertices with number represent edges (see Section 6).

specifics of individual roads. Typical large-scale models describe the topology in terms of length, shape, the locations of intersections, and so on. Some of the models consider other linear features of a road, such as bridges, viaducts, tunnels, and ramps [227]. Topologies are the target of a large-scale model. The sidewalk, driveway, and curb ramp do not need to be distinguished, and other facilities on the road are ignored [227].

The standard description of a street network is a graph $G = (E, V)$, where the edges E represent roads, and the vertices V represent roads' intersections and end-points [44, 215, 216] (referred to as an intersection-based representation). Each vertex has identity information such as coordinate, vertex number or intersection name. Since typical lanes have persistent driving directions, the street network can be modeled as a directed graph. The boundary points (e.g., entrance and exit), length, name, and the speed limit of a road are attached to directed edges. In a traffic network without one-way roads, the model can be simplified as an undirected graph by merging lanes. The traffic network graph is used in the research of vehicle assignment with shareability networks, such as [213, 214, 217, 218]. A shareability network assumes that all users can share network resources and the service provider can globally schedule resources. Under the premise of ensuring the quality to passengers, the shared network can be used to find the optimal resource allocation strategy. While the road network graph models are useful to study the topological properties of the network such as connectivity, closeness, and centrality, they typically simplify some important geometric aspects [80]. And there are some studies dividing cities into grids of equal size. Vertex set V represents regions corresponding to small grids, and edges are used to encode relations between regions [228]. According to adjacency relations, functional similarity and road accessibility between regions, road models are abstracted into three kinds of graphs [228]. Based on the nearest neighbor rule, Voronoi tessellations are used to divide urban areas into variable-size partitions, which is an effective model in a space with uneven data distribution [229].

In contrast, a street network analysis may represent streets as vertices of a graph, while the junctions between streets (i.e., crossings) are the edges [215] (referred to as a street-based representation). We give a comparison in Figure 11, which consists of the map of a street network and its two graph representations. The junction graph (b) is the intersection-based network graph, and the segment graph (c) is the street-based network graph.

Stochastic geometry models represent streets as lines or line segments rather than links in a graph. The Poisson line process (PLP), Poisson-line tessellations (PLT), Poisson-Voronoi tessellations (PVT), and Poisson Delaunay tessellations (PDT) were used to facilitate capturing the topological relationships of roads in [220]. It is shown in [60, 80] that the PLP has some analytical tractability. Other components of an actual road, such as vehicle and road side unit, are placed in the network topology model according to the Poisson distribution. There are abundant practical applications to estimate the cost of a telecommunication access network, deploy of urban infrastructure, and explore the spatial distribution patterns of vehicles [80, 219, 220].

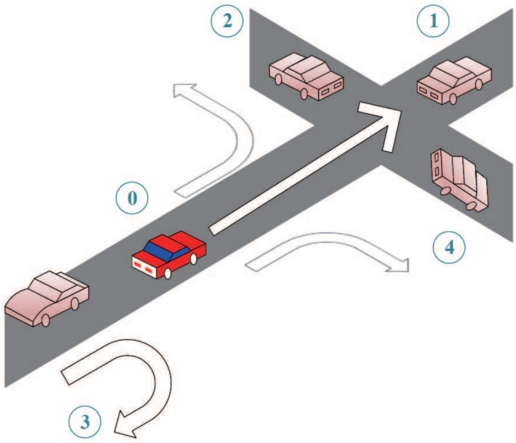


Figure 12 (Color online) Schematic diagram of vehicle movement directions. The five choices are parking (0), going straight (1), turning left (2), turning around (3), and turning right (4) (see Section 6).

6.2 Small-scale road model

There are three basic road-map requirements for intelligent vehicle systems: centimeter-level accuracy, storage efficiency, and usability [221]. Therefore, efficient and reliable digital maps are crucial to the development of ITS applications [222]. Road modeling is a vital part of generating digital maps [223]. A small-scale road model contains detailed lane information, so that it is also valuable for the modeling of driving behavior.

Guo et al. [222] proposed a low-cost solution for the automatic generation of a precise lane-level map by using sensors installed in vehicles. The construction of the map can be divided into two steps, road orthographic image generation, and lane graph construction. The readings of GPS, INS, and visual odometry are often fused to generate a road orthographic image. The lane information is extracted from the road orthographic image and a large number of vehicle trajectories. The lane centerlines are described as smooth curves. To accurately describe the lane information, clothoid splines can be utilized (a clothoid is a spiral whose curvature is a linear function of its arc length), and cubic splines can be employed to describe the normal roadways and virtual transition lanes in a link segment [222]. Typical transition lanes model the paths that pass through intersections.

Gwon et al. [221] used mathematical curve segmentation to describe the lane information (see [221, Eq. 5]). A trajectory $L(s)$ is defined to be a cubic spline curve that is composed of M sequentially connected piecewise polynomial curves $L_m(s), m = 1, \ldots, M$, to achieve storage efficiency of the map. The cardinal spline—a sequence of individual curves joined to form a larger curve—is devised to build an initial road model in [223], and a gradual optimization algorithm to determine the optimal control point and tension parameters.

Matrices can be applied to describe possible combinations of passing ways while analyzing traffic flows at intersections [47]. There are five choices for the movement of the vehicle on the road, as shown in Figure 12. A phase matrix can be used to describe the non-conflicting combinations of four movement types around an intersection. The three types of movements are turning right, going straight, and turning left [47]. A 4-by-4 binary matrix is used to indicate the movement type allowed. For example, Figure 13 describes one movement type. A state of the intersection accounts for a possible combination of the movement type at the intersection, as illustrated in the matrix in Figure 13(b). The transition of the road state can be modeled as a Markov model. It is of interest to analyze the vehicle turning trajectory, to select the appropriate length of collision-free steering combined with a control signal and to optimize traffic flows at intersections, as in [47, 224–226].

Jin et al. [48] introduced the R-tree data structure into the road network graph to represent both stationary and moving targets in a road network, such as a supermarket and a vehicle. The model has two layers. The first layer indicates the edge in the road network. The second layer consists of five R-trees, four of which are used to store moving objects on the edge and the fifth to store stationary objects. The two-layer structure is mapped by a hash table. An example is shown in Figure 14, which is the storage structure of Figure 11(a). The graph and two-layer structure can be updated in real-time to provide a spatial-temporal query service.

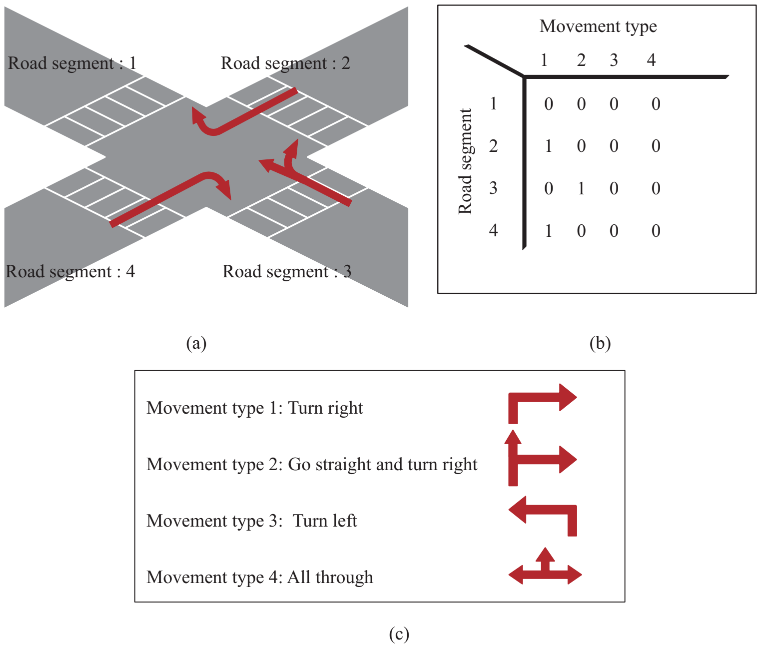


Figure 13 (Color online) Movement types and phase matrices. In (b), '1' means 'permitted' and '0' means 'not permitted'. The implementation of phase matrix (b) at the intersection is shown in (a) (see Section 6). (a) Illustration of the four road segments of an intersection; (b) one example of phase matrix; (c) illustration of the four movement types.

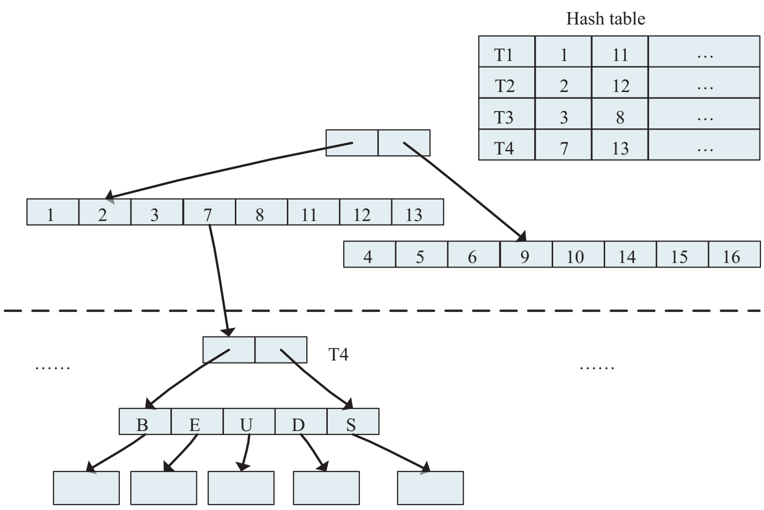
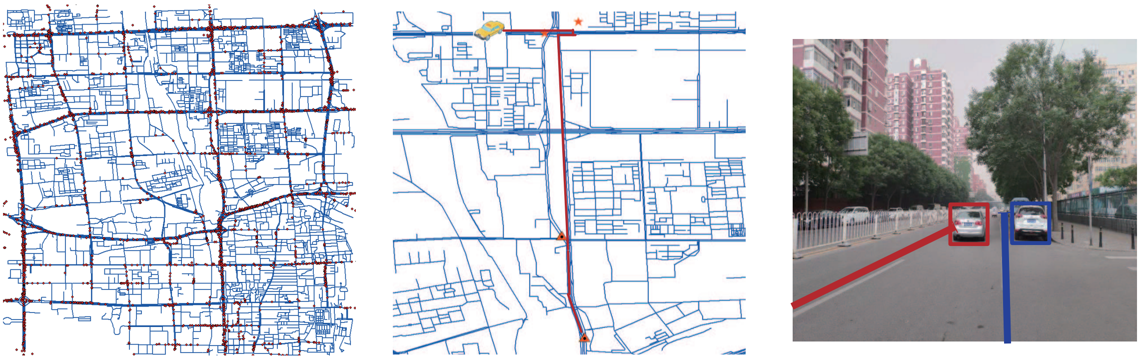


Figure 14 (Color online) Data structure of the objects in city road network. The first level R-tree represents the edges in city road network. The second level R-trees store the objects on the edges. The hash table is used to find the corresponding R-Tree based on the edge ID. The number represents the ID of edges. 'T4' represents the ID of the R-tree. In the second level R-tree, objects are divided into moving objects (B, E, U, D) and static objects (S), in which 'B' represents objects moving from the start point, 'E' represents objects moving from the end point, 'U' represents objects moving upward from a point between the start point and the end point, and 'D' represents objects moving backward from a point between the start point and the end point (see Section 6).

There are several ways to model lanes, for example, the clothoid road model (for a planar road) and B-spline-based road model (for 3D lane recognitions) [230]. The Kalman filter is used to track these lane models for autonomous driving on a highway. A lane-level road model of an intersection was proposed in [231], in which the road network is modeled from the perspective of topological characteristics and geometrical characteristics. The topological characteristics of roads include connectivity, turn rules, and properties of the crossing. The geometrical characteristics of roads include the multi-lanes of the internal part of the special intersection which is described as a cardinal spline. The cardinal spline model can approximate practical vehicle trajectories at the intersection well. Comparisons between different small-scale road models are shown in Table 6.



(a) (b) (c)

Figure 15 (Color online) The hierarchy diagram of applications in ITS (see Section 7). (a) Underlying network connectivity based applications; (b) off-line network optimization based applications; (c) real-time automatic drive based applications.

Table 7 The modeling methods used in various applications of ITS (see Section 7)

Scenario	Requirement	Example
Underlying network connectivity related applications	Derive metrics for network connectivity of VANETs; design a practical communication network for ITS.	PPP [58, 67, 68, 78] LGCP [41, 42] PALM [232]
Off-line network optimization related applications	A long-term stable estimate of road traffic pressure; active traffic management; travel demand forecasting.	CTM [233, 234] N-S model [235] Road network model [214, 218, 236–238]
Real-time autonomous driving related applications	Path planning strategy under different optimization targets, autonomous driving/advanced driver-assistance systems.	(Multi-class model combined) [224, 226, 237, 239–249]

7 Specific applications of various models

This section introduces several specific applications of the models. According to our suggested model-driven architecture (as illustrated in Figure 1), the interesting applications are off-line network optimization and prediction, online network functionality, and real-time autonomous driving and prediction, to satisfy different requirements of underlying network connectivity. Examples are shown in Figure 15. Table 7 [41, 42, 58, 67, 68, 78, 214, 218, 224, 226, 232–249] is the modeling methods used in various applications of ITS.

It is useful to analyze the network performance and design a suitable communication strategy. Off-line demands are about the rational construction and efficient use of transportation systems and communication networks. The dynamic allocation of network resources in adaption to traffic demand is critical to V2V, as it can help solve problems, such as information forwarding interrupt in sparse networks and limited access ability in dense networks. Efficient utilization of resources (such as taxis, buses, and roads) is expected to bridge the gap between the high travel demand and limited transportation networks. The active management of traffic flows and the task assignment of urban demand are two aspects of the efficient application. The goals are to achieve a safe and comfortable ride experience. On-line path planning and autonomous driving technologies/advanced driver-assistance systems are concrete applications of the IoV.

7.1 Application for joint network design

The connectivity in the physical layer is closely related to the geometric distribution and the transmission range of the vehicles, and ensures the reliable transmission of information in VANETs [29, 73]. By analyzing the connection performance based on stochastic geometry and queueing theory, routing algorithms are proposed to increase system throughput and reduce network delay. One example is provided in Figure 15(a), in which the blue lines represent the road map of Beijing and the red dots indicate the location of the taxis at 8:00 am on November 5, 2012.

In the past decade, quite a few studies have analyzed the connectivity of VANETs based on stochastic geometry. The general research approach proceeds as follows.

Step 1. The spatial distribution of vehicles on a road is based on an underlying mathematical model.

The arrival of vehicles on highways is often modeled as a Poisson process [67]. The Cox point process is mostly used [41, 60] in urban areas.

Step 2. A practical traffic network is developed by combining the spatial distribution with dynamic traffic flow characteristics (such as the relationship between vehicle density and speed) or with other traffic elements (such as the traffic lights and intersection) [66].

Step 3. Based on the traffic network model, metrics of network connectivity [250], such as the average space headway, the cluster lengths/sizes, the degree of a node, the probability of being disconnected, and the network capacity, are derived.

Step 4. The above metrics capture the communication capabilities of the VANETs and help derive a better communication strategy and deployment plan of the infrastructure [41, 42].

Ref. [58] studied the constant-intensity traffic flow under the assumption that the communicating vehicles are independent of each other and form a uniform Poisson point process. One method of inter-vehicle communication on highways is the inter-vehicle forwarding communication (a relay process). The transmission distance of homogeneous/heterogeneous traffic satisfies the gamma distribution. And the relationship between propagation distance, vehicle (with relay forwarding capability) density, and transmission range are discussed in [58].

Ref. [68] amended the factors of robustness to the exponential distribution of the headway to represent an unstable traffic network with disturbance. An expression for VANETs connectivity considering dynamic traffic flow characteristics (average speed headway, average space headway, and the variance of space headway) is obtained. They prove that vehicular mobility has an impact on VANETs connectivity. For vehicles arriving as a PPP, Ref. [69] introduced the influence of the traffic parameters (distribution of speed and the traffic flow) and the vehicular transmission distance on VANETs' connectivity. By using stochastic ordering techniques, Ref. [69] transformed the connection distance distribution problem into a busy period distribution problem of the equivalent infinite server queue.

The actual data reveals that vehicle spacing satisfies the exponential distribution as long as communicable vehicles' density falls below a certain value (for example, 1000 vehicles/h) [78]. Based on this conclusion, the characteristics of sparse VANETs were studied in [78], including the probability of being disconnected from the following vehicle, the cluster size, the cluster lengths, and the intra- and inter-cluster spacing. Ref. [78] explored and quantified the re-healing time of disconnected networks. Stochastic geometry is a useful tool for designing the routing protocols and determining the effectiveness of applications in a network with partitions.

Assuming that the arrivals of vehicles at highway entrances follow a Poisson process, Khabazian et al. [67] obtained the mean cluster size and the probability that a newly arriving node or a random node will see the entire node population in a single cluster at a steady state. Then, the connectivity of multi-lane highway is analyzed. The authors of [73] added spatial randomness of the receiving vehicles into a path loss model and developed a triple composite Fox channel model. The dual randomness of V2V communication is evaluated by using two indicators: signal error probability and channel capacity.

By analyzing actual taxi trajectory data, Cui et al. [41, 42] proved that the LGCP model matches precisely with the spatial distribution of vehicles in large and small cities. The V2V connectivity is analyzed under the lognormal channel model, and the node degree is used to measure network connectivity [42]. The connectivity and capacity of VANETs were analyzed by using stochastic geometry in [250]. A closed-form expression is derived for the probability distribution of the number of disconnected links. The study of the network disconnection degree is of great help in accurately estimating the number of cluster heads or mobile relays. Ref. [250] provided a reference for the allocation of network resources for potential VANET-LTE inter-operability.

For the network design problem, the method of operations research only establishes a detailed geographic network description model [77]. Random graph theory can transform these spatial features into macroscopic laws and obtain a network cost function for optimizing network construction [77]. The intention of the design of the spatial point process model for road systems is to analyze the communication network architectures on highways, as was pointed out in [77].

Developing new energy vehicles (such as electric vehicles) is an important direction in the deployment of smart cities. It is promising to guide the deployment of infrastructure, such as charging piles on highways, by analyzing the spatial characteristics of vehicles [232]. The conservation law of vehicles can capture arrival behaviors, and the stochastic PALM can describe charging demand. The combination model can identify the spatial and temporal dynamics of electric vehicle charging demand. This can facilitate optimizing the arrangement of charging stations and smart-grid load.

7.2 Application for off-line network optimization

The traffic flow models cover various control and optimization problems in traffic management. For different application requirements, it may be necessary to combine models with different levels of detail. Vehicle mobility has an important impact on both transportation and communication networks [251]. Active traffic management can achieve efficient utilization and assignment of transportation and communication network resources.

An application example is shown in Figure 15(b), in which the blue lines represent the actual traffic route of Beijing, the stars indicate two pick-up locations, and the triangles indicate the destinations of two passengers. Scheduling an autonomous fleet is a solution for efficient utilization of resources while preserving user experience. The red track is the trajectory of the taxi service for two passengers. The implementation method can be referred to as [218]. Long-term infrastructure maintenance planning was proposed in [233]. The CTM is used to characterize the traffic flow. The cells in CTM are considered as an infrastructure element (such as based stations, roads, bridges, and guardrails) that needs to be maintained. One can optimize maintenance plan by balancing the user loss and the maintenance cost through a mixed-integer bi-level program.

The second application is active traffic management. The phase transition between a jamming phase and a flow phase occurs at, or near, the point of the maximum throughput of the traffic [152]. Practitioners can effectively control the traffic flow and increase network efficiency by forecasting the maximum throughput point of urban. Esser and Schreckenberg [235] developed an N-S model simulation tool for urban traffic, which has been tested in the inner city of Duisburg, Germany. The network dynamics is obtained from the N-S simulation model, including vehicle motion, signal light update, and data acquisition statistics. A traffic signal control method is formulated as a mixed integer program based on the CTM [234].

The next one is resource allocation and task assignment such as travel demand forecast [252–254]. Predicting the state of road traffic to guide effective travel is a significant application. Cui et al. [236] modeled the road system as a queueing network and solved the vehicle flow using network calculus. Based on the research of the traffic flow characteristics, they proposed an ML method to find the non-blocking speed threshold and the road capacity for every region. This method can achieve an on-line road-level route planning by avoiding the saturated/congested sections. This work minimizes the travel time by maintaining the speed in a route by an energy-efficient way.

Also, Cui et al. [236] designed a task assignment via bipartite graph matching, and the optimization goal is to minimize the waiting time. The minimum fleet problem and optimal task allocation scheme of the city are also promising applications. The minimum fleet of urban areas is the optimal number of vehicles required by municipalities and taxi companies to provide efficient services [214, 218, 237, 238]. A solution has been found in [214], and the current number of vehicles can be reduced by half to meet the demand.

7.3 Applications for online network functionality

Data-driven approaches with appropriate machine learning and inference techniques have been adopted to develop the online algorithm of anticipatory mobility management to achieve ultra-low latency networking [255, 256]. These approaches have the potential to enable proactive communication for future vehicular networks.

One potential application of interest is proactive network association with ultra-low latency in open-loop downlink transmissions. Ref. [256] developed a prediction method of access requests based on big vehicular data analysis techniques which can be used in a function design of ultra-low latency mobile networks. Another interesting application is online latency management. A latency measurement method was proposed in [257] in which the Bayes filter is used to obtain real-time prediction of AP locations. Ref. [258] put forth a latency management method which predicts whether the current connection status can meet the latency requirements of mobile services in a vehicular network. Accordingly, adjustment of the connections can be carried out, if needed.

7.4 Application for real-time autonomous driving

Real-time prediction and simulation can be used to study behaviors of vehicles during driving, as shown in Figure 15(c) [259, Figure 2]. As discussed earlier, a lot of researches have been carried out on the driving behavior to capture the car following, lane changing, and acceleration overtaking. By studying the car's following behaviors, the sophisticated systems can be innovated by predictively controlling the driving behavior of the leading car in a platoon. Advanced driver-assistance systems and autonomous driving draw a lot of interest in academic and engineering applications. The development of these applications depends on the ultra-reliable low-latency communication environment to relay information.

7.4.1 *Trip planning*

Vehicular traffic models can reveal the movement pattern and forecast the most economical trajectory by considering characteristic information (social status, habits of life, identity characteristics, and so on) and mining historical data. For example, decisions can be made based on historical movement patterns in a mixed traffic condition with both pedestrians and vehicles [260].

Some studies such as [224,226,242] endeavored to design intelligent traffic light systems. The intelligent traffic light system can properly extend green light time, by monitoring and predicting incoming vehicles, and can contribute to, e.g., system throughput and the average waiting time. With the popularity of networked autonomous vehicles, intelligently scheduling vehicle resources plays a more and more important role in practical applications. Beyond trip planning, the data-driven approach can be applied for fleet management of autonomous vehicles in the smart city [236, 261]. A future traffic state inference mechanism is established based on taxi historical track, order data set, and real-time traffic information set, and the path planning of taxi picking up passengers is proposed according to the inference results [262].

7.4.2 *Autonomous driving/driver-assistance system*

With the increasing popularity of in-vehicle networks and the sensor deployments, it is easy to obtain a lot of real-time information about the state of vehicles, such as speed, direction, engine state, and charging state. The sensitive information obtained from these sensors would provide strong support for modeling and performance analysis. Ref. [263] reviewed a collision avoidance system. It is designed to reduce the possibility of an accident by using different sensors (radar, laser, and camera). Ref. [264] proposed a model predictive control (MPC)-based shared steering framework for intelligent vehicles. Under this cooperative steering framework, the reliability of drivers is analyzed in dangerous situations and in the predictive time domain. On this basis, two improved schemes are proposed, which can reduce the vehicle state oscillation and enhance the safety of intelligent vehicles. A new sensing device was presented in [265], which can monitor traffic congestion and urban flash floods by real-time vehicle detection, classification, and speed estimation in the context of wireless sensor networks. The planar light detection and ranging sensor are used to detect obstacles. The authors of [183] formulated a multi-phase optimal control problem to simultaneously optimize the reference speed and steering angle within the detection range. Except for vehicle sensors, smartphones and roadside sensors also bring broader prospects of application [40,266,267]. These developments have brought new opportunities for the development of autonomous driving/assisted driving systems.

The key problems that autonomous driving/driver-assistance system needs to solve include scene recognition, lane detection, traffic sign detection, etc. [268]. The deep learning method is used for scene identification, and a context deconvolution network is designed, in which channel and spatial context modules process global and local features, respectively [269]. Ref. [270] proposed a kind of CNN combining self-attention and channel attention for lane marker detection. The neural network based on CNN was used to detect traffic signs, which will extract the features of the region of Interest, and then Softmax classifier was used for classification [271].

The advantage of human driving is the driver's rich experience and judgment in dealing with various emergencies. The disadvantage of manual driving is that the behaviors of drivers are often affected by the environment, physical condition, and psychological state, possibly causing an error of judgment and excessive reaction time. One idea of autonomous driving is to adapt to specific situations dynamically by imitating human behavior. The studies about human-centric intelligent driver assistance systems were abundant [39,259,272–275]. Ref. [276] combined the predicted driver behavioral information of the vehicle with surrounding information for braking assistance and warning the driver in time. Busso et al. [277] predicted the perceived distraction of the drivers in both visual and cognitive by training regularized regression models.

The adaptive cruise control (ACC) system is a type of assisted driving. The ACC system is applied to control the car-following behavior that is not easily disturbed on highways [243]. Many manufacturers

Table 8 The comparison of related work (see Section 8)

Survey	Year	Scope	Categorization criterion	Categorization
[11]	1998	Macroscopic traffic flow models	Order of mathematical equations	First-order models, higher order models
[12]	2001	Vehicular traffic flow models	Level-of-detail of features	(Sub)Microscopic/mesoscopic/macroscopic models
[19] [20]	2009	Vehicular mobility models	Vehicular motion patterns generate techniques	Synthetic models, survey-based models, trace-based models, traffic simulator-based models
[14]	2012	Car-following models	Definition of safety distance	Relative motion of vehicles based models, status of preceding vehicles based models
[15] [16]	2013	Lane-changing models	Lane changing decision or impact	Equations based models artificial intelligence models
[13]	2014	Vehicular traffic flow models	Level-of-detail of features	Microscopic models, mesoscopic models, macroscopic models
[17]	2014	Motion prediction and risk assessment models	Degree of abstraction	Physics-based motion models, maneuver-based motion models, interaction-aware motion models
[18]	2016	Driver behavior models	Desired use of models	Models for lane changing, intersection decision making, driver profiling, and travel assistance
[279]	2017	Vehicle headway distribution models	Historical developments	Headway distribution models in 1930s–1970s, 1970s–1990s, and 1990s–present
[50]	2018	Data-driven short-term traffic prediction	Vehicles spatial correlation	Temporal dependence models, temporal-spatial dependency models
This survey	2022	Vehicular mobility models	The perspective of application requirements	Network connectivity based models, network optimization based models, autonomous driving based models

have introduced the ACC system to practical applications, such as Google and Tesla. Recently, the research of [243] used the ACC system for plug-in hybrid electric vehicles to develop an ecological adaptive cruise controller (Eco-ACC). Aiming at achieving energy-efficiency, they use a nonlinear model predictive control technology (NMPC) to optimize the vehicle velocity. An ACC system with lane changing by coordinating with the driver's operation was developed in [245]. The ACC system is responsible for the vertical operation and risk assessment, and the driver operates the steering wheel to achieve horizontal operation.

An advanced driver-assistance system (ADAS) [249] is developed to prevent rear-end crashes. The method can monitor the distance of the front vehicle and warn the driver of an imminent collision. The ADAS can assist drivers' conduction of lane changing. The driver-assistance system (DAS) of [247] can adjust the longitudinal and latitudinal acceleration during a lane-changing process. For reliable autonomous driving, it is important to understand the dynamic characteristics of surrounding vehicles and estimate the potential risk of mixed traffic. An intention identification model (IIM) by employing the LSTM networks was developed to identify the intention of traffic participants in the surrounding environment [278].

Ref. [248] proposed a scenario model predictive control (SCMPC) using the data-driven CFM and LCM to predict the trajectories of the vehicles. Scenario-based models represent uncertainty using samples and do not require a priori probability distributions for their predictions [248]. The uncertainty of the environment is described by the scenario-based algorithm, which combines the MPC method with the traffic prediction model. The system, combining the CFM of the drivers with the dynamic model of the surrounding vehicles, assesses the risk of lane changing. Then, the MPC is used to achieve dual-target tracking and smooth switching. A cascaded fuzzy inference system (CFIS) was proposed in [246]. The CFIS measures the distance and the speed relative to the vehicle in front of the target adjacent lane by the spread spectrum radars. It provides a more reasonable, safer, and more comfortable ride experience [246].

Remark. The developed ACC system is still limited to highways with smooth roads. To achieve autonomous driving, the introduction of real road topology and real-time trajectory prediction is critical.

8 Existing surveys

The relevant research has been summarized in Table 8 [12–20, 50, 279]. Papageorgiou [11] discussed the macroscopic traffic model, where it is pointed out that the theoretical modeling is of significance to understanding and reproducing traffic flows. However, empirical verification is the standard for measuring the accuracy of the model [11]. Hoogendoorn et al. [12] classified the vehicular traffic model according to the level of details (such as (sub)microscopic, mesoscopic, and macroscopic models), and discussed the applicability according to the accuracy of the models. It was claimed in [12] that the microscopic model is suitable for off-line simulation, and the gas dynamic model is the basis for deriving a fluid model. The fluid dynamics model is suitable for estimation, prediction, and control of flows. The development and the trend of traffic flow models are shown in the form of a model tree in [13]. Wageningen-Kessels et al. [13] also gave several application examples of traffic flow models.

The authors of [19,20] refined the classification criteria, covering the various levels of motion (e.g., trip, path, and flow). In general, the macro- and microscopic division methods in [12] are in use. Härri et al. [20] analyzed the characteristics of the models from the perspective of network and application. And the interaction between the mobility model and the network simulator was discussed in [19]. Härri et al. [19,20] provided a framework for generating realistic vehicular mobility patterns, which is of significance to guide the study of the relationship between application requirements and motion module selection.

In [14], the development of CFM was reviewed. The full velocity difference (FVD) model is an example to resolve the local stability and asymptotic stability of the model. LCMs used in a computer simulation were reviewed in [15]. Rahman et al. [15] compared various models from the perspective of the lane-changing decision, the reason for lane changing, target lane selection, gap acceptance and whether the diver variability is considered. The LCMs were classified into the lane-changing decision-making processes and the impacts of lane changing on surrounding traffic in [16].

Based on the degree of abstraction, the motion prediction and risk assessment models for intelligent vehicles are classified [17]. The computational complexity and real-time performance of trajectory prediction methods were compared in [17]. Authors of [18] reviewed driver behavior models designed in an attempt to improve in-vehicle and smartphone sensing and communication capabilities. The latest research [50] reviewed the data-driven traffic prediction approaches and presented several future directions for traffic forecast. Li et al. [50] divided forecast models into time-dependent predictive models and space-dependent predictive models.

As discussed above, some studies reviewed the vehicular mobility modeling status, and some guided the future direction of modeling. However, there is little research on the classification of existing models from the perspective of applications and requirements. It is a complex task to select a suitable model from a large number of models to achieve a specific application. Also, the existing surveys of vehicular mobility models introduce few studies about the past 10 years. We provide a detailed survey of vehicular mobility models aiming at helping to solve problems in ITS scenarios. This contribution presents a comprehensive review of the popular models in the past 10 years. This study analyzes the application scenarios of the models. It can provide the references and guidance for the selection of models in the complex application scenarios of ITS collated in Figure 1.

9 Open challenges

Efficient integration of modeled features (and feature extraction) and deep (reinforcement) learning platforms is still missing, despite its prosperity. Existing learning platforms typically driven by big data [163–165], are transiting to be driven by models—model-driven learning [6]. However, existing learning platforms are yet to be ready to take in the model paraments describing the vehicular modeling of a particular segment of a road, or a section of a city, as features to produce useful inferences and predictions. Nevertheless, the recent advances in vehicular models and deep learning have provided the way towards a comprehensive integration of both.

Further compressing the modeled features by extracting features from different models (or models from different perspectives) can further reduce the input to learning platforms, while maintaining or even improving the learning effectiveness and efficiency. In this sense, it could be of practical interest to pre-fuse or pre-synthesize the modeled features (with different perspectives); or, in other words, to develop hybrid models of vehicular mobility to capture the different aspects and their correlation. The

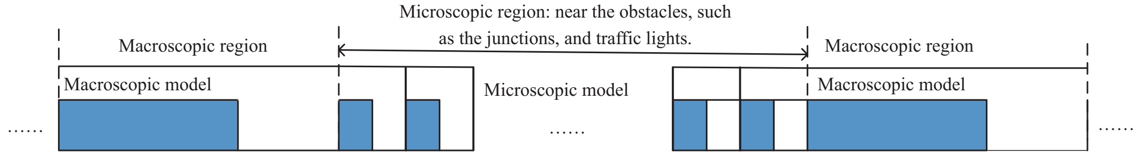


Figure 16 (Color online) Hybrid Lagrangian model (see Section 9).

hybrid features can be submitted to the learning platforms.

There are early attempts to integrate different models capturing different features in the vehicular context. For example, neural networks work together with a nonparametric regression model to improve prediction accuracy [129]. Differential equation models are combined with empirical models to overcome a backward travel problem in differential equation traffic flow models [280, 281]. Other joint or hybrid models were presented in [251, 282–284]. The joint models can typically perform better by combining nonparametric models with kinetic models. Ref. [251] modeled vehicular mobility by integrating the effects of both transportation and communication networks. Combinations of the discrete phase transition with the continuum model were taken in [251] to solve a negative speed problem of high-order differential equation models. Phase transition phenomena can be predicted based on the continuous flow models [280, 285–287]. Rascle et al. [280] proposed a hybrid model based solely on a Lagrangian discretization of both the ARZ model and the car-following models. As shown in Figure 16 [280, Figure 3], an integration of a macroscopic model and a microscopic model is able to predict traffic conditions far away from intersections, signal lights, and obstacles. Ref. [281] incorporated the car-following theory into a continuous traffic flow model to overcome the backward travel problem.

Further, the integration of vehicular mobility models and road network models is another important research area. To date, road network models have been studied separately from vehicular mobility models, while vehicular mobility heavily depends on the roads and road conditions. The road network models can be very useful to calibrate the vehicular mobility models for better accuracy. This integration or joint studies of vehicular mobility and road networks can take place in both the time and space domains, and tackle the potentially strong correlations between above mentioned domains. The correlations can contribute to the accurate prediction of traffic in both the time and space domains, and in turn, the predictive assignment of communication resources. More complex relationships between road networks and vehicular mobility deserve further and comprehensive investigation to holistically design joint models of the road networks and vehicular mobility.

10 Conclusion

This study reviewed the status quo of vehicular mobility models with an emphasis on the latest breakthroughs on the three aspects of space and time distribution models of vehicles, vehicular traffic flow models, and driver behavior and fleet pattern. Road models were also discussed, which provided underlying assumption of vehicular mobility models and can be potentially designed jointly with vehicular mobility models.

Capturing and extracting different key features of vehicular mobility, the different models reviewed in the study have the potential to facilitate applying state-of-the-art data-driven deep (reinforcement) learning techniques to optimize access, routing, resource allocation, and network security in large-scale IoV networks. The vehicular mobility models can potentially contribute to the generalization and transfer of trained data-driven deep learning models into new areas where no training data is captured and labeled for the development of the deep learning models.

As also discussed in this study, a number of open challenges are critical to an effective integration of vehicular mobility models/features and deep learning techniques, and yet to be addressed in the literature. The examples include new deep neural network designs capable of digesting multiple modeled vehicular mobility features, and comprehensive hybrid models capturing different vehicular mobility features/aspects and their temporal and spatial correlations. These challenges deserve continuous research effort to bring IoV technologies to fruition.

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