The SINR Meta Distribution in Poisson Cellular Networks

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***Abstract*—This work studies the signal-to-interference-plus-noise ratio (SINR) meta distribution (MD) in cellular networks with a focus on the Poisson model. Firstly, we show that for stationary base station point processes, arbitrary fading, and power-law path loss with exponent α, the base station density λ and the noise power σ^2 impact the SINR MD only through $\eta \triangleq \lambda^{\alpha/2}/\sigma^2$, termed the *network signal-to-noise ratio* (NSNR). Next, we show that for Poisson cellular networks, the SINR MD can be written as $g(x)\theta^{-2/\alpha}$ when the target SINR θ and the target reliability x jointly satisfy a constraint. We derive this constraint and the integral of $g(x)$. Lastly, we discuss several extensions of the results to more general models and architectures.**

***Index Terms*—Cellular networks, SINR meta distributions, stochastic geometry.**

I. Introduction

The meta distribution (MD) of the signal-to-interference-plus-noise ratio (SINR) [1] is an important metric in cellular networks. By separating the randomness over time (small-scale fading) and randomness over space (large-scale path loss), the MD characterizes the network performance more comprehensively than traditional metrics: it gives the entire distribution of the individual link success probability as opposed to its mean. However, the MD is difficult to derive directly using standard stochastic geometry tools. Instead, a common method is to first derive the moments of the link success probability and then calculate the MD or its approximations based on the moments [1]–[3]. A key step in this method is the efficient calculation of the distributions given the moments.

In contrast, this work focuses on the analytical properties of the MD that directly show the impact of key system parameters. Recently, we have shown in [4] that in interference-limited Poisson networks with power-law path loss with path loss exponent α, the signal-to-interference ratio (SIR) MD for arbitrary fading can be expressed as $g(x)\theta^{-2/\alpha}$ when the target SIR threshold θ and the target reliability x jointly satisfy a constraint. Due to the interference-limitedness assumption in [4], noise is neglected. However, depending on the network density and the signal propagation condition, noise can have a significant impact on the MD. This work characterizes such an impact by studying the SINR MD. Specifically, we show that for power-law path loss and arbitrary fading, the noise power σ^2 and the base station (BS) density λ jointly impact the MD only through the *network signal-to-noise ratio* (NSNR), defined as follows.

Definition 1 (Network Signal-to-Noise Ratio)**.** *The network signal-to-noise ratio for a stationary BS point process with density λ is*

$$\eta \triangleq \lambda^{\alpha/2}/\sigma^2. \tag{1}$$

Next, focusing on Poisson networks, we show that the form $g(x)\theta^{-2/\alpha}$ is exact for the SINR MD in the same separable region derived in [4] for the interference-limited setting. We further derive the integral of $g(x)$ over $[0,1]$, which depends on the NSNR and the fading statistics.

In the following section, we consider a baseline model with arbitrary independent fading and power-law path loss. In Section III, we discuss a few extensions of the results to more general settings, including general path loss models, multiple resource blocks with selection combining, the ALOHA model, and BS silencing. Section IV concludes the paper.

II. Baseline Model

A. System Model

Consider a stationary and ergodic BS point process $\Phi \subset \mathbb{R}^2$ with intensity λ and the downlink receiver (user) at the origin o. Let x_i, $i \in \mathbb{N}$, denote the ith nearest BS to o in Φ and $r_i \triangleq \|x_i\|$ denote the distance from o to x_i. Consider power-law path loss with exponent $\alpha > 2$, independent fading $\{h_i\}_{i=1}^{\infty}$, additive white Gaussian noise with variance σ^2 and nearest-base station association (NBA). The BS transmission power P is set to 1. The SINR at the origin is

$$\mathrm{SINR} = \frac{h_1 r_1^{-\alpha}}{\sum_{i=2}^{\infty} h_i r_i^{-\alpha} + \sigma^2},$$

where $\mathbb{E}h_i = 1$. The link success probability is

$$P_{\mathrm{s}}(\theta) \triangleq \mathbb{P}\left(\mathrm{SINR} > \theta \mid \Phi\right), \quad \theta > 0,$$

which is referred to as the conditional success probability (CSP). The SINR MD is [1]

$$\bar{F}_{P_{\mathrm{s}}}(\theta, x) \triangleq \mathbb{P}(P_{\mathrm{s}}(\theta) > x), \quad x \in [0,1],$$

which is parametrized by two variables, namely the target link SINR threshold θ and the target link reliability threshold x. Note that in contrast to bipolar network models [5], adding noise does not reduce the support of the distribution of $P_{\mathrm{s}}(\theta)$. The standard success probability is $\mathbb{E}P_{\mathrm{s}}(\theta) = \int_0^1 \bar{F}_{P_{\mathrm{s}}}(\theta, x)\mathrm{d}x$. For the Poisson point process (PPP) with iid Rayleigh fading, $\mathbb{E}P_{\mathrm{s}}(\theta)$ is calculated in [6, Proposition 7.3.1] and also available in [7]; the moments of the CSP $M_b(\theta) \triangleq \mathbb{E}P_s(\theta)^b$, $b \in \mathbb{C}$, are calculated in [1] for the interference-limited case ($\sigma^2 = 0$) and in [3] for $\sigma^2 > 0$.

B. General Networks

Let Φ_0 be an arbitrary stationary point process of density 1. We assume that the BS locations are modeled by $\Phi = \Phi_0/\sqrt{\lambda}$, where $\lambda > 0$ is the density of Φ.[1]

Lemma 1. *For the baseline model, the* SINR *MD depends on λ and σ^2 only through the NSNR. Equivalently, if the bandwidth (and thus the noise power) is doubled, the BS density needs to be increased by a factor $2^{2/\alpha}$ to maintain the same performance.*

Proof: Let us write

$$\mathrm{SINR} = \frac{h_1}{\sum_{i=2}^{\infty} h_i r_1^{\alpha} r_i^{-\alpha} + r_1^{\alpha}\sigma^2}$$

and

$$P_{\mathrm{s}}(\theta) = \mathbb{P}\bigg(h_1 > \theta\bigg(r_1^{\alpha}\sum_{i=2}^{\infty} h_i r_i^{-\alpha} + r_1^{\alpha}\sigma^2\bigg) \;\bigg|\; \Phi\bigg). \quad (2)$$

For any fading distribution, $r_1^{\alpha}\sum_{i=2}^{\infty} h_i r_i^{-\alpha}$ does not depend on either λ or σ, since $\{r_i\}_{i=1}^{\infty}$ is scaled by the same factor $1/\sqrt{\lambda}$. The distribution of the random variable $r_1^{\alpha}\sigma^2$ depends on $\eta = \lambda^{\alpha/2}/\sigma^2$. ■

We observe that densifying the network has the same effect as reducing the noise power. In the ultradense case, as $\eta \to \infty$, the network becomes noise-free—as if $\sigma^2 \to 0$. Conversely, when $\eta \to 0$, the network is interference-free, but the SINR decays to 0. Lemma 1 shows that two networks that differ only in the BS density and noise power are equivalent in their SINR MD performance if $\lambda_1^{\alpha/2}/\sigma_1^2 = \lambda_2^{\alpha/2}/\sigma_2^2$. One may define a crossover point of η and classify the network as interference-limited if $\eta > 1$ (assuming unit transmission power) and noise-limited otherwise. For $\alpha = 4$, $\eta = \lambda^2/\sigma^2$. So if there is twice as much noise, the BS density needs to be increased by a factor of $\sqrt{2}$ to maintain the same performance.

In a practical setting, relevant network parameters include the BS transmission power P (normalized to 1 in this work), the BS density (estimated by the average inter-site distance), and the noise power (estimated by noise density and channel bandwidth). We can generalize the definition of NSNR to include P, *i.e.*, $\eta = P\lambda^{\alpha/2}/\sigma^2$. For instance, a dense urban scenario may have $\lambda \approx 25/\mathrm{km}^2$ ($\lambda \approx 1/d^2$ for inter-site distance $d = 200$ m in a square lattice) and noise power -94 dBm (for noise density -174 dBm/Hz and 100 MHz channel bandwidth) [8], [9].

A similar observation to Lemma 1 is made in [10, Lemma 4] for the standard success probability, *i.e.*, $\mathbb{E}P_{\mathrm{s}}(\theta)$, and only for the PPP. Note that the equivalence in the standard success probability does not imply the equivalence in the SINR MD, while the opposite holds trivially since $\mathbb{E}P_{\mathrm{s}}(\theta)$ is obtained by integrating the MD over the parameter x. For instance, it is known that for Poisson networks with instantaneously-strongest base station association (ISBA), $\mathbb{E}P_{\mathrm{s}}(\theta)$ does not depend on the fading distribution. Thus it is equivalent to the success probability for no fading [11]. Such an equivalence does not hold for the SINR MD: $P_{\mathrm{s}}(\theta) \in \{0, 1\}$ for no fading while $0 < P_{\mathrm{s}}(\theta) < 1$ for general fading (with continuous distribution).

[1]This way, if Φ_0 is a hard-core process with hard-core distance u, Φ has a hard-core distance $u/\sqrt{\lambda}$.

C. Poisson Networks

We now derive the separability of the SINR MD in Poisson networks for arbitrary fading. Later, we consider two special cases of fading: no fading and Rayleigh fading. We use the dual interpretation of the SINR MD [12] to obtain the rate distribution for fixed reliability. Then we discuss the impact of the fading statistics.

1) Separability: Define $\mathbb{R}^+ \triangleq [0, \infty)$ and $\delta \triangleq 2/\alpha$. The following theorem characterizes the separability of the SINR MD in Poisson cellular networks.

Theorem 1. *For Poisson networks, there exists a function $g : [0, 1] \to \mathbb{R}^+$ such that the SINR MD is*

$$\bar{F}_{P_{\mathrm{s}}}(\theta, x) = g(x)\theta^{-\delta}, \quad (\theta, x) \in \mathcal{D},$$

where

$$\mathcal{D} \triangleq \{(\theta, x) : \mathbb{P}(h_1/h_2 > \theta) \le x\}, \quad (3)$$

and g depends on the distributions of the fading random variables $\{h_i\}_{i\in\mathbb{N}}$ and η. Further, g is monotonically decreasing with $g(1) = 0$.

Proof: Let us rewrite (2) as

$$P_{\mathrm{s}}(\theta) = \mathbb{P}\bigg(h_1 > \theta\Big(\frac{r_1}{r_2}\Big)^{\alpha}\bigg(h_2 + \sum_{i=3}^{\infty} h_i\Big(\frac{r_2}{r_i}\Big)^{\alpha} + r_2^{\alpha}\sigma^2\bigg) \;\bigg|\; \Phi\bigg). \quad (4)$$

$P_{\mathrm{s}}(\theta)$ is continuous and monotonically decreases with $\theta r_1^{\alpha}/r_2^{\alpha}$. Denote the inverse of $P_{\mathrm{s}}(\theta)$ w.r.t. $\theta r_1^{\alpha}/r_2^{\alpha}$ as f. f is a function of the random variables $\{r_i\}_{i\ge 2}$ and depends on σ^2. Then

$$\begin{aligned}\bar{F}_{P_{\mathrm{s}}}(\theta, x) &= \mathbb{P}(P_{\mathrm{s}}(\theta) > x) \\ &= \mathbb{P}(\theta r_1^{\alpha}/r_2^{\alpha} < f(x, r_2, \ldots)) \\ &= \mathbb{E}[\mathbb{P}(\theta r_1^{\alpha}/r_2^{\alpha} < f(x, r_2, \ldots) \mid r_2, \ldots)] \\ &= \mathbb{E}\Big[\min\Big\{1, \theta^{-\delta} f(x, r_2, \ldots)^{\delta}\Big\}\Big].\end{aligned}$$

The last step follows from the fact that for a PPP, $\mathbb{P}(r_1/r_2 < t) = t^2$ and r_1/r_2 is independent from $\{r_i\}_{i\ge 2}$ [4].

For $(\theta, x) \in \mathcal{D}$,

$$P_{\mathrm{s}}(\theta) > x \quad \Rightarrow \quad P_{\mathrm{s}}(\theta) > \mathbb{P}(h_1 > \theta h_2) \quad (5)$$

Writing $P_{\mathrm{s}}(\theta)$ as (4), we conclude $f(x, r_2, \ldots) < \theta$ for $\forall\{r_i\}_{i\ge 2}$, $(\theta, x) \in \mathcal{D}$. Note that $\sum_{i=3}^{\infty} h_i(r_2/r_i)^{\alpha} + r_2^{\alpha}\sigma^2 > 0$. Hence

$$\begin{aligned}\bar{F}_{P_{\mathrm{s}}}(\theta, x) &= \theta^{-\delta}\mathbb{E}\Big[f(x, r_2, \ldots)^{\delta}\Big] \quad (6) \\ &= \theta^{-\delta} g(x), \quad (\theta, x) \in \mathcal{D}.\end{aligned}$$

The dependence of g on the fading distributions is obvious, and the dependence on η follows from Lemma 1. The monotonicity of g is inherited from the monotonicity of the SINR MD with $\bar{F}_{P_{\mathrm{s}}}(\theta, 1) = 0$ for any $\theta > 0$. ■

When $(\theta, x) \notin \mathcal{D}$, (6) becomes an upper bound. Adding noise does not change the separable region in [4]. This is because the noise power, which appears in an extra term $r_2^{\alpha}\sigma^2$ in (4), does not change the regime in which $f < \theta$.

Remark 1. *From (3), the separable region is $x \geq 0$ as $\theta \to \infty$. So $\lim_{\theta\to\infty} \theta^\delta \mathbb{E}P_{\rm s}(\theta) = \lim_{\theta\to\infty} \theta^\delta \int_0^1 \bar{F}_{P_{\rm s}}(\theta, x){\rm d}x = \lim_{\theta\to\infty} \theta^\delta \int_{\mathcal{D}} g(x)\theta^{-\delta}{\rm d}x = \int_0^1 g(x){\rm d}x$. For $\sigma = 0$, $\int_0^1 g(x){\rm d}x = {\rm sinc}(\delta)$ where ${\rm sinc}(x) \triangleq \sin(\pi x)/(\pi x)$. This follows from [13, Theorem 4] which shows that for the PPP with arbitrary iid fading, $\mathbb{E}P_{\rm s}(\theta) \sim {\rm sinc}(\delta)\theta^{-\delta}$, $\theta \to \infty$. Theorem 1 shows that $\mathbb{E}P_{\rm s}(\theta) = \Theta(\theta^{-\delta})$ also holds for Poisson networks with noise, and the pre-constant depends on the noise. To obtain the pre-constant $\int_0^1 g(x){\rm d}x$ for $\sigma > 0$, we can directly modify the statement and proof in [13, Theorem 4] to include noise. For the PPP with iid fading h,*

$$\mathbb{E}P_{\rm s}(\theta) \sim {\rm sinc}(\delta)\theta^{-\delta} \int_0^\infty \exp(-x - Kx^{1/\delta}){\rm d}x, \ \theta \to \infty, \tag{7}$$

where $K \triangleq (\pi\mathbb{E}[h^\delta]\Gamma(1-\delta))^{-1/\delta}/\eta$. Hence for $\sigma > 0$, $\int_0^1 g(x){\rm d}x$ depends on $\mathbb{E}[h^\delta]$. Specifically, for $\delta = 1/2$,

$$\int_0^1 g(x){\rm d}x = \frac{1}{\sqrt{\pi K}}{\rm erfc}\left(\frac{1}{2\sqrt{K}}\right)\exp\left(\frac{1}{4K}\right). \tag{8}$$

For no fading, $K = 1/(\eta\pi^3)$, and for Rayleigh fading, $K = 4/(\eta\pi^4)$. Further, [13, Theorem 4] proves that $\mathbb{E}P_{\rm s}(\theta) = \Theta(\theta^{-\delta})$ holds for all simple stationary BS point processes in the noise-free scenario. A natural conjecture is that the MD satisfies $\Theta(\theta^{-\delta})$ for all simple stationary BS point processes with noise.

Remark 2. *For iid Nakagami-m fading [4, Theorem 2],*

$$\mathcal{D} = \left\{(\theta, x) : I_{\frac{1}{1+\theta}}(m, m) \leq x\right\},$$

where $\theta > 0$, $x \in [0, 1]$, and $I_p(a, b)$ is the regularized incomplete beta function. Let $\mathcal{D}^\square$ denote the separable region expressed in terms of $(1/(1+\theta), x) \subset [0, 1]^2$. For any iid fading, $\mathcal{D}^\square$ always contains the point $(1/2, 1/2)$, and the area of $\mathcal{D}^\square$ is $1/2$ [4, Corollary 1]. For independent but non-identical fading, $\mathcal{D}^\square$ may not contain $(1/2, 1/2)$. For instance, for $h_1 \equiv 1$ and h_2 exponentially distributed, $\mathcal{D} = \{(\theta, x) \colon \theta \geq -1/\log(1-x)\}$, and the area of $\mathcal{D}^\square$ is $1 + e\,{\rm Ei}(-1) \approx 0.404$. ${\rm Ei}(x) = -\int_{-x}^\infty \frac{\exp(-t)}{t}{\rm d}t$ is the exponential integral.

2) No fading: Without fading, *i.e.*, $h \equiv 1$, $P_{\rm s}(\theta) \in \{0, 1\}$. $\bar{F}_{P_{\rm s}}(\theta, x) = \mathbb{E}P_{\rm s}(\theta) = \mathbb{P}({\rm SINR} > \theta)$, $\forall x \in (0, 1)$. Applying Theorem 1, the separable region is $\mathcal{D} = \{\theta \geq 1, 0 < x < 1\}$, and $\mathbb{P}({\rm SINR} > \theta) = g(x)\theta^{-\delta}$, $\theta \geq 1$. For $x \in (0, 1)$, $g(x) \equiv g$ is a constant.

$$g = {\rm sinc}(\delta) \int_0^\infty \exp(-x - Kx^{1/\delta}){\rm d}x, \tag{9}$$

where $K = (\pi\Gamma(1-\delta))^{-1/\delta}/\eta$. For $\delta = 1/2$, $K = 1/(\eta\pi^3)$, and g can be written in a closed-form as (8). Eq (9) simplifies the expression derived in [10, Corollary 5] and generalizes [11, Corollary 2] which considers the noise-free scenario.

3) Rayleigh fading: With iid Rayleigh fading, the CSP is [3]

$$P_{\rm s}(\theta) = \exp(-\sigma^2\theta r_1^\alpha) \prod_{i=2}^\infty \frac{1}{1 + \theta\left(r_1/r_i\right)^\alpha}. \tag{10}$$

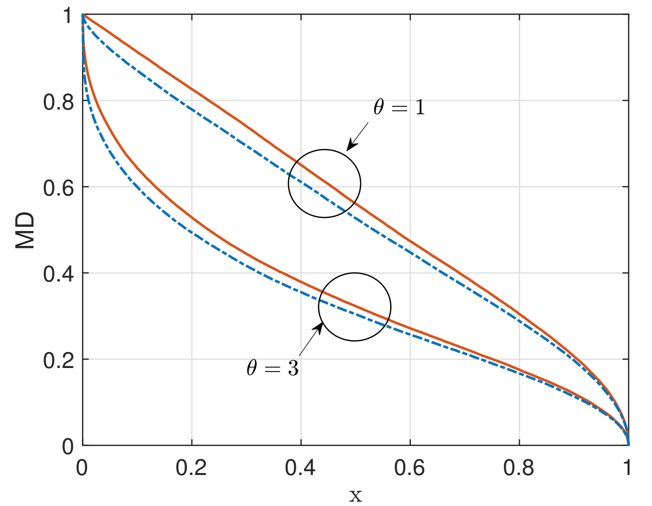

Fig. 1. MD vs. x for $\theta = 1$ and $\theta = 3$, iid Rayleigh fading, $\delta = 1/2$. The solid curves are for $\eta = \infty$ and the dashed curves are for $\eta = 1$.

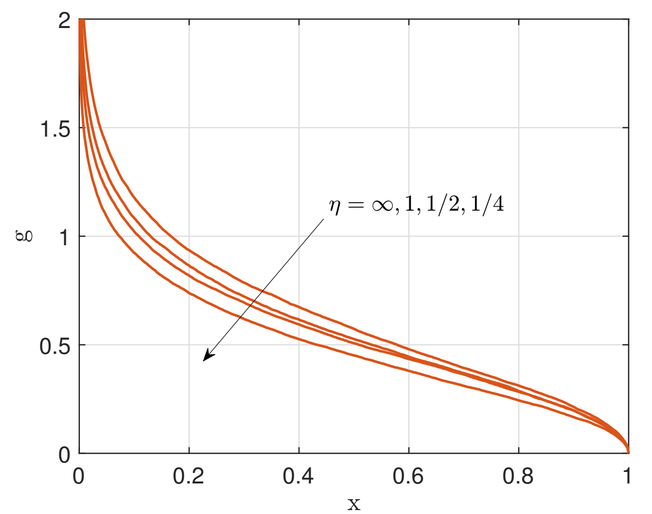

Fig. 2. Simulation results of $g(x)$ for $\eta = \infty, 1, 1/2, 1/4$, iid Rayleigh fading, $\delta = 1/2$.

Using the probability generating functional (PGFL) of the PPP, its b-th moment is derived in [3]. Applying Theorem 1, we know that $\bar{F}_{P_{\rm s}}(\theta, x) = g(x)\theta^{-\delta}$ for $\mathcal{D} = \left\{(\theta, x) : 1 + \theta \geq x^{-1}\right\}$. $g(x)$ can be obtained through a quick simulation and/or approximated by analytical expressions using the same methods as in [4]. We obtain $g(x)$ by simulating $\bar{F}_{P_{\rm s}}(100, x)100^\delta$. Fig. 1 shows the impact of η on the MD for SINR thresholds $\theta = 1$ and $\theta = 3$. Fig. 2 shows the impact of η on g.

4) Dual interpretation: Given Φ, let $T(x)$ be the SINR threshold such that the link achieves a target reliability x, *i.e.*, $P_{\rm s}(T(x)) \equiv x$. We have

$$\mathbb{P}\left(P_{\rm s}(\theta) > x\right) \equiv \mathbb{P}\left(T(x) > \theta\right). \tag{11}$$

This is known as the dual interpretation of the MD [12]. Thus, for a fixed reliability x, the link SINR threshold is Pareto distributed when $(\theta, x) \in \mathcal{D}$. Theorem 1 provides a good characterization for the link rate distribution over its entire range when a reasonably high reliability is targeted. Fig. 3 shows the link SINR threshold distribution for $x = 0.9$ and

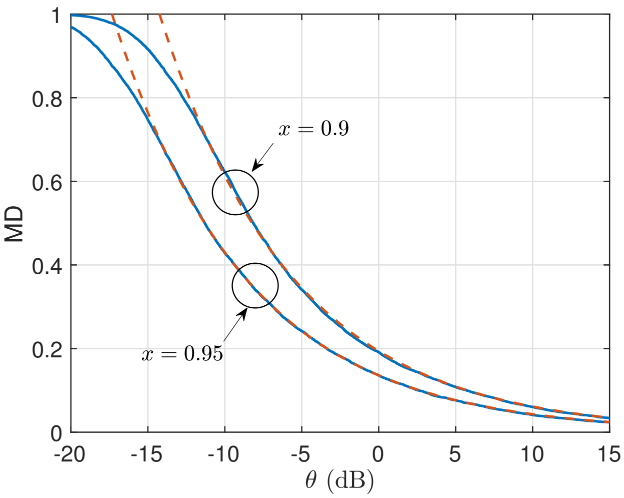

Fig. 3. Link threshold SINR distribution for fixed reliability $x = 0.9$, $x = 0.95$, iid Rayleigh fading, $\eta = 1$, $\delta = 1/2$.

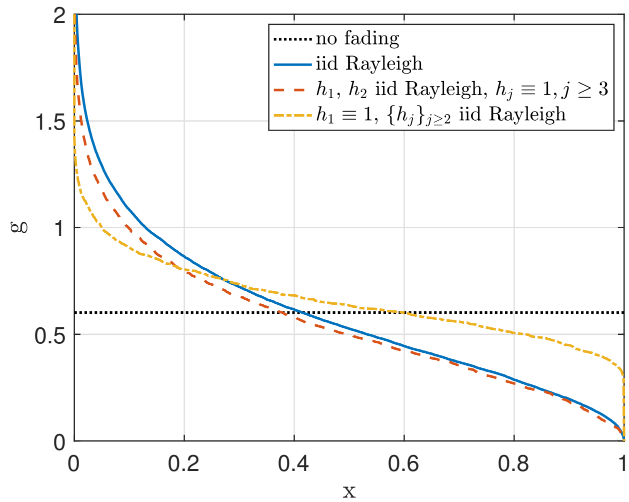

Fig. 4. $g(x)$ for four fading scenarios. $\eta = 1$, $\delta = 1/2$.

$x = 0.95$ with iid Rayleigh fading, $\eta = 1$. $g(0.9) = 0.197$ and $g(0.95) = 0.137$. The form $g(x)\theta^{-\delta}$ is exact for $\theta \geq -9.54$ dB and $\theta \geq -12.79$ dB, respectively. For $\eta = 1$, 59% of the users achieve an SINR of -9.54 dB or higher with reliability 0.9. In contrast, 63% achieve the same performance when $\eta = \infty$.

5) Impact of Fading: To show the impact of the fading statistics on g, we compare the following four fading scenarios: *i)* no fading, *ii)* iid Rayleigh fading, *iii)* h_1, h_2 iid Rayleigh fading, $h_j \equiv 1$, $j \geq 3$, and *iv)* $h_1 \equiv 1$, $\{h_j\}_{j\geq 2}$ iid Rayleigh fading. For *i)*, $\mathcal{D} = \{\theta \geq 1\}$, and $g(x)$ is a constant for $x \in (0, 1)$ as expressed in (9), which is approximately 0.6017 for $\delta = 1/2$, $\eta = 1$. For the remaining scenarios, $g(x)$ is obtained through simulation. For scenarios *ii)* and *iii)*, $\mathcal{D} = \left\{(\theta, x) : 1 + \theta \geq x^{-1}\right\}$. For *iv)*, $\mathcal{D} = \left\{(\theta, x)\colon \theta \geq -1/\log(1-x)\right\}$, as noted in Remark 2. The results are plotted in Fig. 4. $g(x)$ for *iii)* is lower than that for *ii)* since the CSP for *iii)* is smaller than that for *ii)* given the same BS locations. However, the difference is minor, which shows that the fading statistics of faraway interferers do not significantly impact g. In contrast, $g(x)$ for *iv)* is significantly larger than *ii)* at the high-reliability end, and lower at the low-reliability end. Hence combating multi-path fading from the serving BS is crucial for the high reliability scenario and less so for the low reliability scenario.

III. Extensions

In this section, we discuss several extensions to more general settings.

A. General Path Loss Models

Consider a general path loss model $\ell(r)$, with the only requirement that it decreases with the distance r and has an inverse ℓ^{-1}. In general, the tail distribution of the SINR MD in terms of θ depends critically on the path loss from the serving BS, and we can modify the proof for Theorem 1 for ℓ. Consider the special case where $h \equiv 1$ (no fading). For the PPP and $\theta \geq 1$,

$$\mathbb{P}(\text{SINR} > \theta) = \mathbb{P}\left(\ell(r_1) < \theta\left(\sum_{i=2}^{\infty}\ell(r_i) + \sigma^2\right)\right)$$
$$\stackrel{\text{(a)}}{=} \mathbb{P}\left[\frac{r_1}{r_2} < \frac{\ell^{-1}\left(\theta\left(\sum_{i=2}^{\infty}\ell(r_i) + \sigma^2\right)\right)}{r_2}\right].$$

Step (a) follows from first inverting ℓ and then dividing r_2. $\theta \geq 1$ guarantees $\ell^{-1}\left(\theta\left(\sum_{i=2}^{\infty} l(r_i) + \sigma^2\right)\right) < r_2$. If $\ell^{-1}\left(\theta\left(\sum_{i=2}^{\infty} l(r_i) + \sigma^2\right)\right) > 0$, we can apply $\mathbb{P}(r_1/r_2 < x) = x^2$ and the conditional independence of r_1/r_2 given $\{r_i\}_{i\geq 2}$. The power-law path loss satisfies $\ell^{-1}(xy) = \ell^{-1}(x)\ell^{-1}(y)$, so $(\ell^{-1}(\theta))^2$ will be extracted from the right hand side of (a).

B. General Path Loss With Selection Combining

Let $n \in \mathbb{N}$ be the number of resource blocks used. We assume the fading coefficients from each BS across different resource blocks are iid. The resource block with the maximum SINR is chosen to decode the message. We have

$$P_{\mathrm{s}}^{(n)}(\theta) = 1 - (1 - P_{\mathrm{s}}(\theta))^n, \tag{12}$$

and

$$\bar{F}_{P_{\mathrm{s}}}^{(n)}(\theta, x) = \bar{F}_{P_{\mathrm{s}}}\left(\theta, 1 - (1-x)^{\frac{1}{n}}\right). \tag{13}$$

By the monotonicity of $\bar{F}_{P_{\mathrm{s}}}(\theta, x)$ w.r.t. x, $\bar{F}_{P_{\mathrm{s}}}^{(n)}(\theta, x) \geq \bar{F}_{P_{\mathrm{s}}}^{(m)}(\theta, x)$ if $n \geq m$. For all links to achieve the same target reliability of x, the number of resource blocks n assigned to each link must adapt to the link SINR. Allowing $n \in \mathbb{R}$ to be a link-dependent random variable for fixed target θ and x, we obtain $n = \log(1-x)/\log(1 - P_{\mathrm{s}}(\theta))$.

C. Power-Law Path Loss With ALOHA

Let $\chi_i \sim \text{Bernoulli}(p)$, $i \in \mathbb{N}$, be iid denoting the transmission status of a BS, *i.e.*, x_i is active if $\chi_i = 1$ and inactive otherwise. The CSP is

$$P_{\mathrm{s}}(\theta) = \mathbb{P}\left(\frac{h_1 r_1^{-\alpha}}{\sum_{i=2}^{\infty} h_i \chi_i r_i^{-\alpha} + \sigma^2} > \theta \mid \Phi\right).$$

It is easy to show that Lemma 1 applies to this setting. For the PPP, $\bar{F}_{P_{\rm s}}(\theta,x) = g(x)\theta^{-\delta}$, $(\theta,x) \in \mathcal{D}$, where $\mathcal{D} = \left\{(\theta,x) : pI_{\frac{1}{1+\theta}}(m,m) \le x+p-1\right\}$. This is calculated by

$$\mathbb{P}\left(h_1/h_2 > \chi\theta\right) = \mathbb{E}\left[I_{\frac{1}{1+\theta\chi}}(m,m)\right] = pI_{\frac{1}{1+\theta}}(m,m)+1-p. \tag{14}$$

If $1-x \ge p$, $\mathcal{D}=\emptyset$. The separable region is different from the baseline model with BS density λp, since the separable region in the baseline model does not depend on BS density. For $\sigma=0$, the moments of the CSP are derived in [1].

D. Power-Law Path Loss With BS Silencing

Let $n \in \mathbb{N}$ be the number of silenced BSs. The CSP is

$$P_{\rm s}(\theta) = \mathbb{P}\left(\frac{h_1 r_1^{-\alpha}}{\sum_{i=n+2}^{\infty} h_i r_i^{-\alpha} + \sigma^2} > \theta \mid \Phi\right). \tag{15}$$

Lemma 1 applies to this scenario also. For the PPP, one can derive the separable region via the definition in Theorem 1. $\mathbb{P}(r_1/r_{n+2} \le x) = 1-(1-x^2)^{n+1}$, $x \in [0,1]$ [14]. Thus the MD is $1-\mathbb{E}\left[\left(1-f(x,r_{n+2},r_{n+3},...)\theta^{-\delta}\right)^{n+1}\right]$ in the separable region. For instance, when $n=1$, *i.e.*, the nearest interferer is silenced, the MD can be written as $a(x)\theta^{-\delta} - b(x)\theta^{-2\delta}$ in the separable region. It satisfies $\bar{F}_{P_{\rm s}}(\theta,x) = \Theta(\theta^{-\delta})$, $\theta \to \infty$, and is upper bounded by $1-\left(1-\mathbb{E}f(x,r_{n+2},r_{n+3},...)\theta^{-\delta}\right)^{n+1}$ due to the convexity of power functions on $\mathbb{R}^+$.

IV. Conclusions

This work shows that the noise power and the BS density jointly impact the SINR MD only through the NSNR. For Poisson networks, the SINR MD can be written in form of $g(x)\theta^{-\delta}$ for $(\theta,x) \in \mathcal{D}$. The fading statistics of the serving BS significantly impacts g especially in the high-reliability regime. We also discuss a few extensions where simple analytical results may be obtained. For future work, one interesting question is if the NSNR can be generalized in capturing the effects of noise and BS density in settings such as heterogeneous networks, networks with traffic dynamics, and other path loss models (e.g., LoS/NLoS). It is also worth studying the distributions/variances of the CSP under the ISBA scheme for different fading models in Poisson networks, where the mean success probability does not depend on the fading. Further, the asymptotics of the SINR MD for $\theta \to 0$ and $x \to 1$ are important, due to their relevance to ultra-reliable and low-latency networks and massive machine-type communication.

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