16.6 Parametric surfaces

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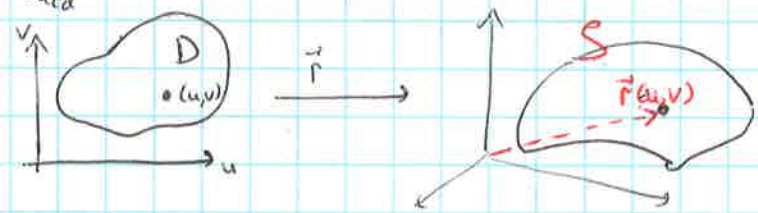
- a curve can be described by $\vec{r}(t)$
- a surface can be described by $\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v)\rangle$
  - vector valued function of u, v on a region D in uv-plane

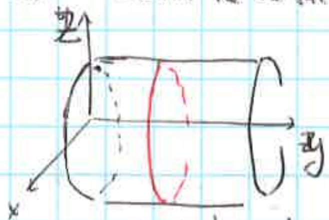
$S:\ x = x(u,v),\ y = y(u,v),\ z = z(u,v)$ — parametric surface

$\underbrace{\qquad\qquad}_{\text{parametric equations}}$



Ex: $\vec{r}(u,v) = \langle 2\cos u, v, 2\sin u\rangle$ sketch the parametric surface

Sol: $x = 2\cos u$, $y = v$, $z = 2\sin u$ $\rightarrow x^2 + z^2 = 4$

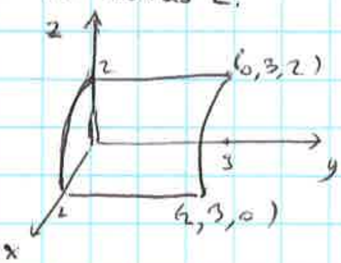


- cylinder of radius 2 along y-axis

vertical slices parall to xz-plane are circles of radius 2.

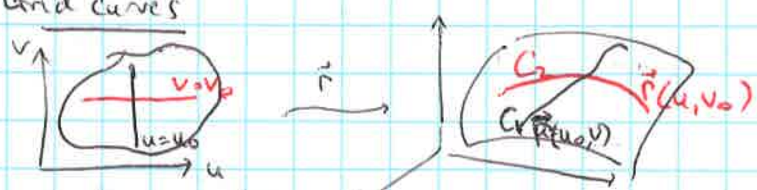
Ex' Imposing restrictions $0 \le u \le \frac{\pi}{2}$, $0 \le v \le 3$, we get

$\rightarrow x, z \ge 0$

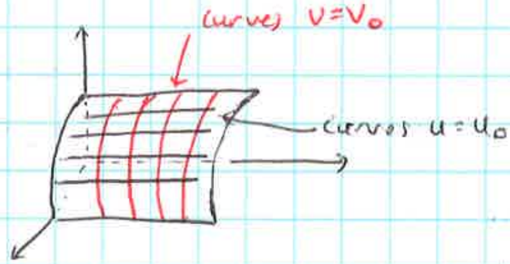


- quarter-cylinder

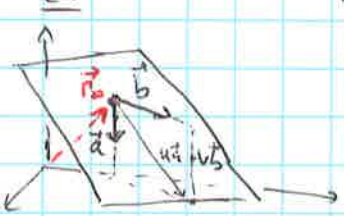
Grid curves



in Ex':



Ex find a vector function representing the plane through P_0 (pos. vector $\vec{r}_0$) containing non-parall vectors $\vec{a}, \vec{b}$.



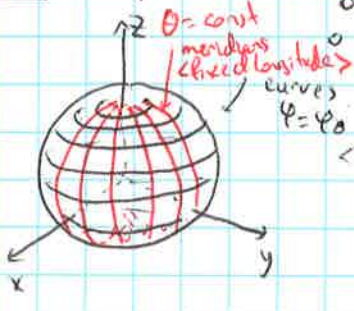
Sol: $\vec{r} = \underbrace{\vec{r}_0}_{\langle x_0,y_0,z_0\rangle} + u\underbrace{\vec{a}}_{\langle a_1,a_2,a_3\rangle} + v\underbrace{\vec{b}}_{\langle b_1,b_2,b_3\rangle}$ or

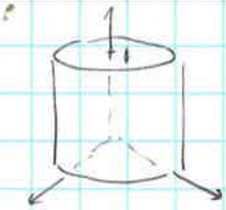
$x = x_0 + a_1 u + b_1 v$
$y = y_0 + a_2 u + b_2 v$
$z = z_0 + a_3 u + b_3 v$

Ex: find a parametric representation of a sphere $x^2 + y^2 + z^2 = a^2$

Sol: in spherical coord: $\rho = a$

$\left.\begin{array}{l} 0 \le \varphi \le \pi \\ 0 \le \theta \le 2\pi \end{array}\right\}$ parameters $\rightarrow$ $\boxed{\begin{array}{l} x = a\sin\varphi\cos\theta \\ y = a\sin\varphi\sin\theta \\ z = a\cos\varphi \end{array}}$ or $\vec{r}(\varphi,\theta) = \langle a\sin\varphi\cos\theta, a\sin\varphi\sin\theta, a\cos\theta\rangle$



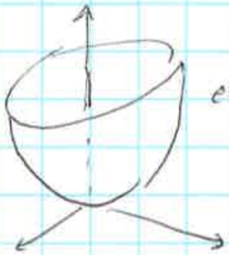


Ex: find a parametric representation of the cylinder $x^2+y^2=4$, $0 \le z \le 1$

Sol: in cylindrical, the cylinder: $r=2$

choose θ, z as parameters.

$$x = 2\cos\theta, \quad y = 2\sin\theta, \quad z = z, \qquad 0 \le \theta \le 2\pi, \quad 0 \le z \le 1$$



elliptic paraboloid

Ex: parametrize the surface $z = x^2+2y^2$

Sol: choose $x=u$, $y=v$ then $\vec{r}(u,v) = \langle u, v, u^2+2v^2 \rangle$

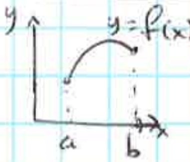
- can do it for any graph of a function $z=f(x,y)$ $\rightsquigarrow \vec{r}(u,v) = \langle u, v, f(u,v) \rangle$

Ex cone $z = 2\sqrt{x^2+y^2}$ or $z^2 = 4x^2+4y^2$, $z \ge 0$ - parametrize

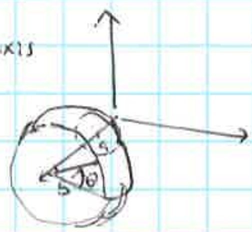
Sol 1: $x=u$, $y=v$, $z=2\sqrt{u^2+v^2}$

Sol 2 choose r, θ as parameters $x = r\cos\theta$, $y = r\sin\theta$, $z = 2\sqrt{x^2+y^2} = 2r$; $\vec{r}(r,\theta) = \langle r\cos\theta, r\sin\theta, 2r \rangle$ with $r \ge 0$, $0 \le \theta \le 2\pi$

Surfaces of revolution



$y=f(x)$ $\xrightarrow{\text{rotate about x-axis}}$



let θ be the angle of rotation about x-axis, then

$$x = x, \quad y = f(x)\cos\theta, \quad z = f(x)\sin\theta$$

$a \le x \le b$, $0 \le \theta \le 2\pi$ - parameters.

Ex: $\boxed{y = \sin x}$, $0 \le x \le 2\pi$ $\longrightarrow$ surface of revolution about x-axis:

$$x = x, \quad y = \sin x \cos\theta, \quad z = \sin x \sin\theta$$