

14.1 Functions of several variables

9/11/2018

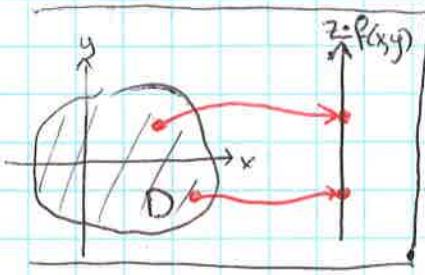
function of two variables

$$f: (x,y) \mapsto f(x,y)$$

in $D \subset \mathbb{R}^2$ in \mathbb{R}

domain

range: set of all values $\in \mathbb{R}$ of $f(x,y)$

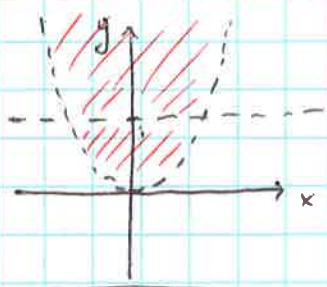


Ex: $f(x,y) = \log\left(\frac{\ln(y-x^2)}{y-1}\right)$

a) find $f(3,1)$
b) find the domain D

Sol a) $f(3,1) = \frac{\ln(3-1^2)}{3-1} = \frac{\ln 2}{2}$

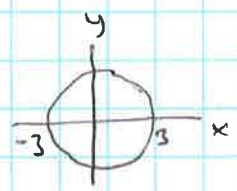
b) domain = all (x,y) s.t. $f(x,y)$ is well-defined, i.e. $D = \{(x,y) \mid y-x^2 > 0, y-1 \neq 0\}$



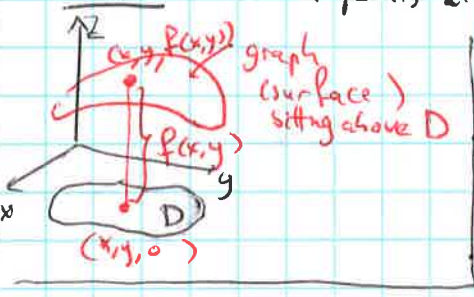
Ex: $g(x,y) = \sqrt{9-x^2-y^2}$ find domain, range

Sol: domain: $9-x^2-y^2 \geq 0$ or $x^2+y^2 \leq 9$

range: $\{z \mid z = \sqrt{9-x^2-y^2}, (x,y) \in D\} = [0,3]$
↳ in $[0,9]$

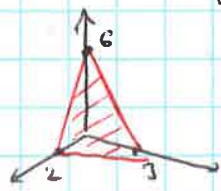


Graph: set of points (x,y,z) in \mathbb{R}^3 with $(x,y) \in D$, $z = f(x,y)$



Ex: $f(x,y) = 6-3x-2y$; sketch the graph

Sol: $z = 6-3x-2y$ or $3x+2y+z = 6$ - plane. Intercepts: $y=z=0 \rightarrow x=2$
 $x=z=0 \rightarrow y=3$
 $x=y=0 \rightarrow z=6$

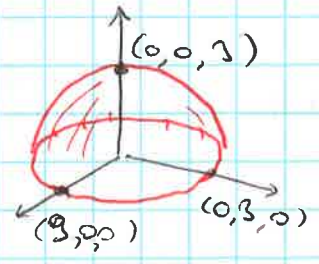


Generally: $f(x,y) = ax+by+c$ - linear function, graph - a plane $z = ax+by+c$

Ex: Graph $g(x,y) = \sqrt{9-x^2-y^2}$

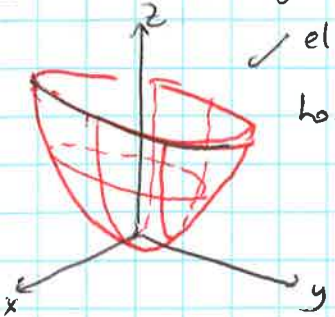
Sol: $z = \sqrt{9-x^2-y^2} \rightarrow z^2 = 9-x^2-y^2 \rightarrow x^2+y^2+z^2 = 9$
also: $z \geq 0$

sphere, $r=3$
upper hemisphere



Ex: $h(x,y) = 4x^2+y^2$. find domain, range, graph

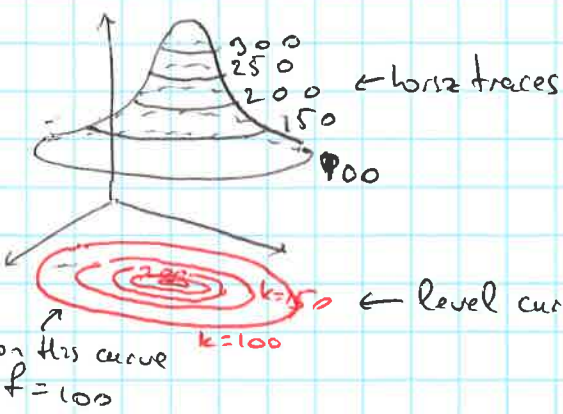
Sol: $D = \mathbb{R}^2$, range: $[0, \infty)$



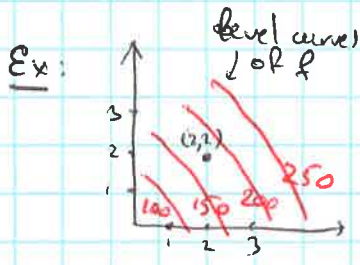
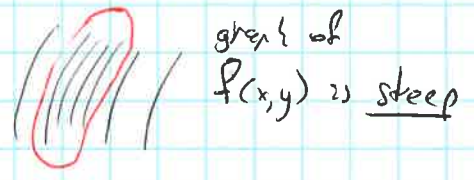
elliptic paraboloid
horizontal traces (cross-sections) - ellipses
= intersection with $z=a$

vertical traces - parabolas

Level curves: curves with eq. $f(x,y) = k$
(or contour) constant in range



where level curves are dense

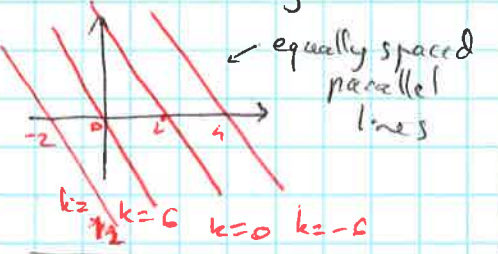


Estimate $f(2,2)$

Sol: $f(2,2) \approx 175$

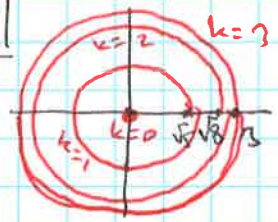
Ex: $f(x,y) = 6 - 3x - 2y$; sketch level curves for $k = -6, 0, 6, 12$

Sol: $6 - 3x - 2y = k$ or $3x + 2y + (k-6) = 0$ - lines with slope $3/2$



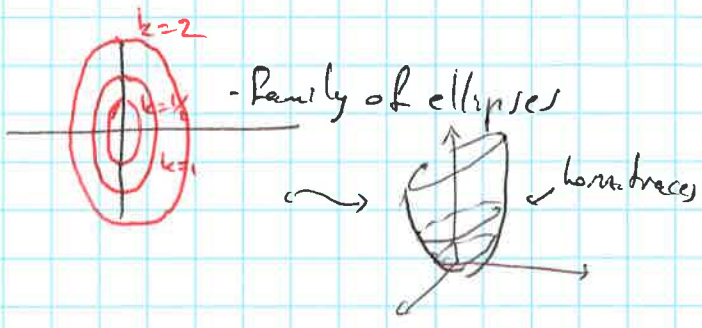
Ex $g(x,y) = \sqrt{9 - x^2 - y^2}$, sketch level curves for $k = 0, 1, 2, 3$

Sol: $\sqrt{9 - x^2 - y^2} = k \rightarrow 9 - x^2 - y^2 = k^2 \rightarrow x^2 + y^2 = 9 - k^2$ - circle of radius $\sqrt{9 - k^2}$



Ex $h(x,y) = 4x^2 + y^2$

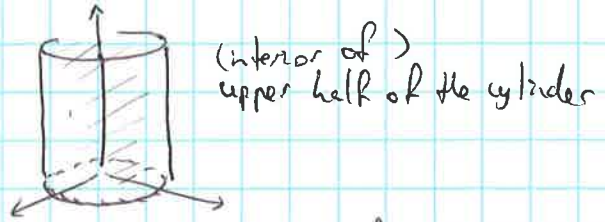
level curves: $4x^2 + y^2 = k \rightarrow \frac{x^2}{1/4k} + \frac{y^2}{k} = 1$



$f(x,y,z)$ - function of 3 variables

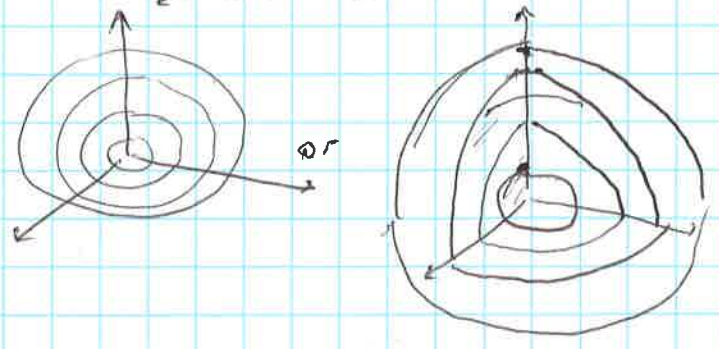
Ex: $f(x,y,z) = \sqrt{1 - x^2 - y^2} + \ln z$ find domain D

Sol: D: $x^2 + y^2 \leq 1, z > 0$



Ex: $f(x,y,z) = x^2 + y^2 + z^2$ find level surfaces

Sol: $x^2 + y^2 + z^2 = k$ ← concentric spheres



optional