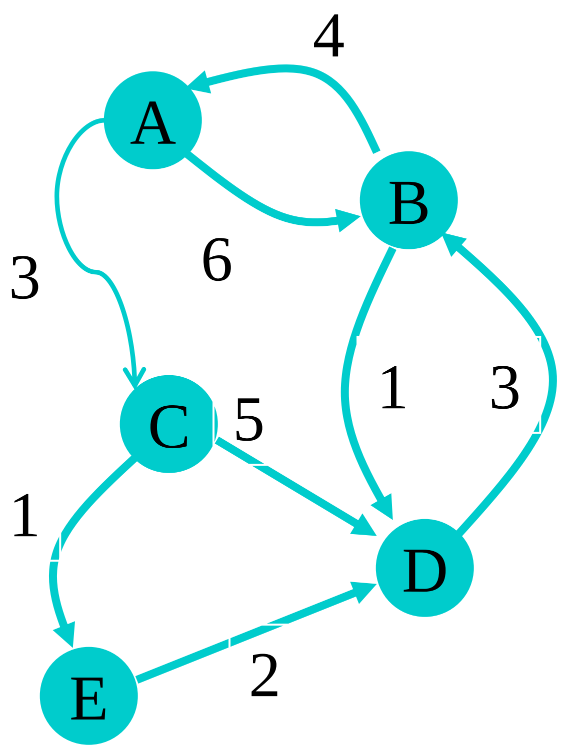
Floyd’s Algorithm

All-pairs Shortest Path Problem

Weighted directed graph:
Let vertices be cities, directed edges be the route traveling from one city to the other, and the weight be time spent on an edge.
All-pairs shortest path problem: find the length of the shortest path between every pair of vertices.



	A	**B**	**C**	**D**	**E**
A	0	6	3	∞	∞
B	4	0	∞	1	∞
C	∞	∞	0	5	1
D	∞	3	∞	0	∞
E	∞	∞	∞	2	0

Representation of the graph as adjacency matrix.
Element (i,j) is the weight of the edge from vertex i to vertex j.

Floyd's Algorithm

```
for k ← 0 to n-1
    for i ← 0 to n-1
        for j ← 0 to n-1
            a[i,j] ← min (a[i,j], a[i,k] + a[k,j])
        endfor
    endfor
endfor
```

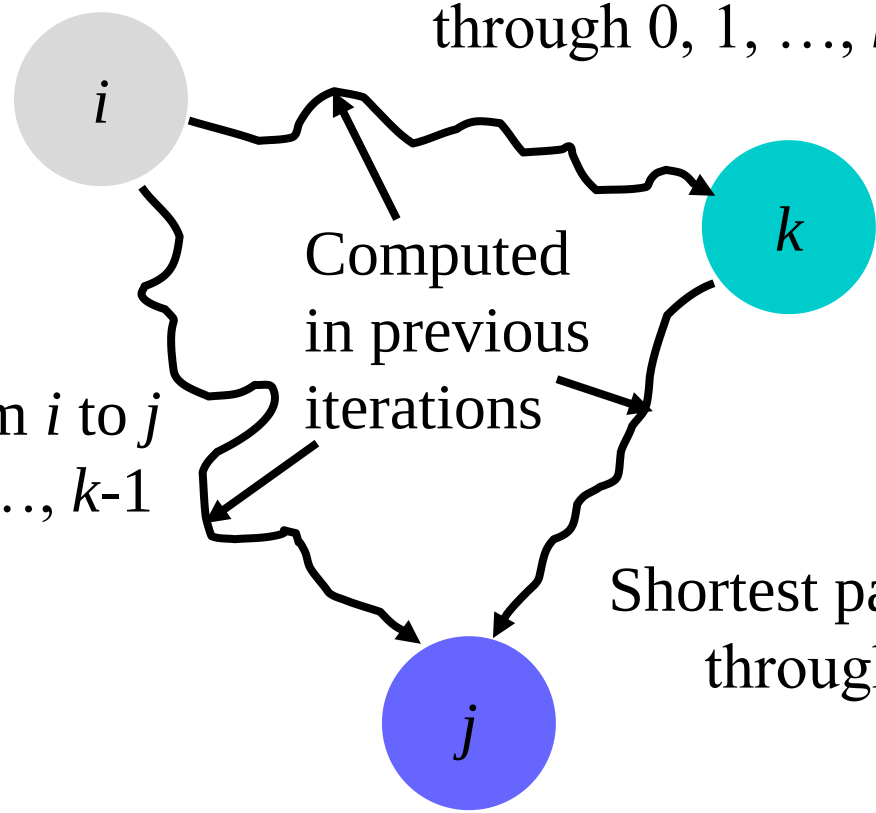
Result

	A	**B**	**C**	**D**	**E**
A	0	6	3	6	4
B	4	0	7	1	8
C	12	6	0	3	1
D	7	3	10	0	11
E	9	5	12	2	0

Solution to the all-pairs shortest path problem. Element (i,j) represents the length of the shortest path from vertex i to vertex j.

How Floyd’s Algorithm Works

Shortest path from *i* to *k* through 0, 1, …, *k*-1



Computed in previous iterations

Shortest path from *i* to *j* through 0, 1, …, *k*-1

Shortest path from *k* to *j* through 0, 1, …, *k*-1

Parallel Algorithm

1. Partitioning
2. Communication
3. Mapping

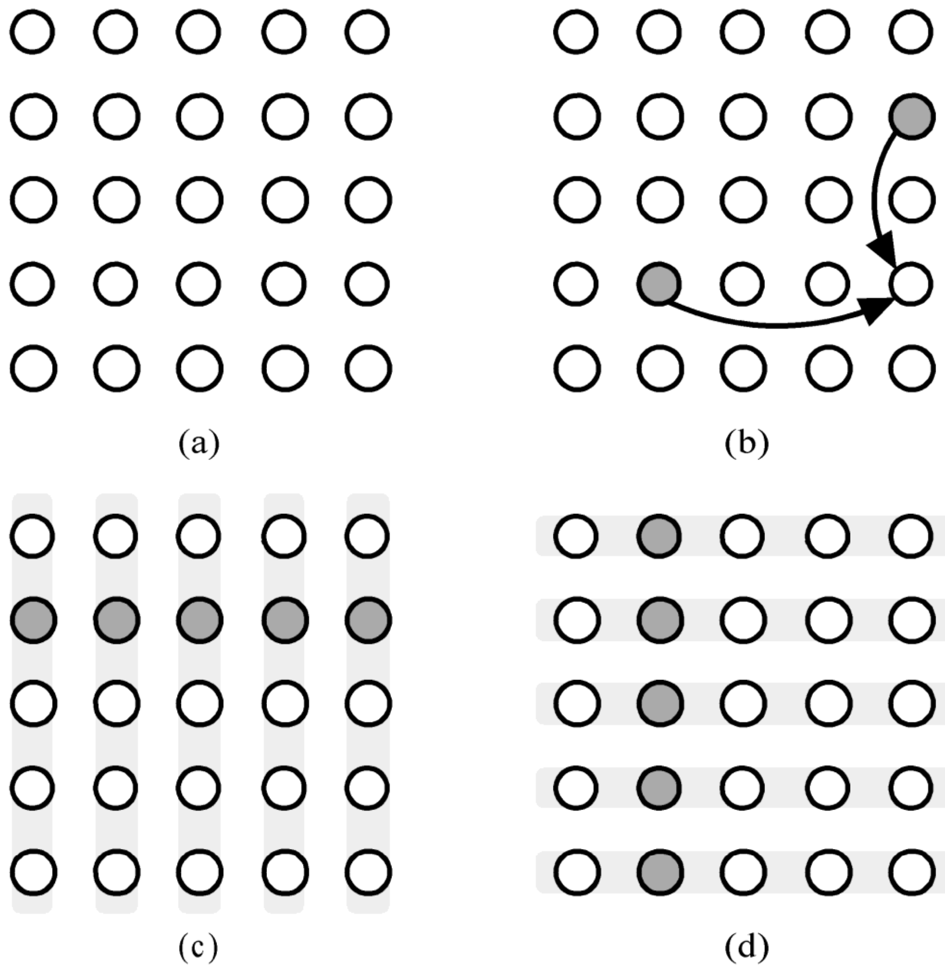
Partitioning

1. Same assignment statement executed n^3 times

2. Domain decomposition: divide matrix **A** into its n^2 elements

Communication

(a) Primitive tasks is associated with each element of the distance matrix

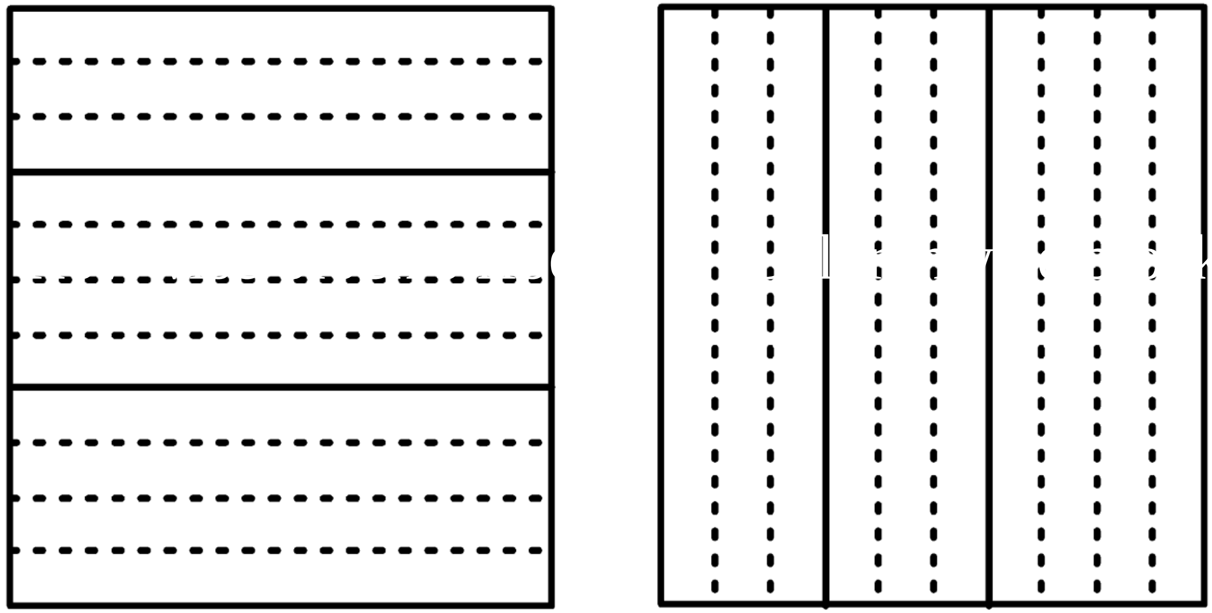


(b) Updating a[3,4] when k = 1. Need old a[3,4] and a[3,1] and a[1,4]

In iteration k: every task in row k broadcasts its value w/in task column

(d) In iteration k: every task in column k broadcasts its value w/in task row

Row/Column Data Decompositions



(a) (b)

1. Column-wise block: Broadcast within columns eliminated
2. Row-wise block: Broadcast within rows eliminated

Expected Things

1. Pseudo code describing the parallel algorithm
2. Justification of choosed communication mode
2. Complexity analysis

   What's the computational complexity (assume a hypercube network for simplicity)?

   What's the communication cost?
3. Performance table

Processes	Execution time (sec)
1	
2	
3	
…	

Reference:

1. R.W. Floyd. Algorithm 97: Shortest path. *Communication of the ACM* 5(6):345, 1962.

2. Ian. Designing and Building Parallel Programs: Concepts and Tools for Parallel Software Engineering. Reading, MA: Addison-Wesley, 1995.