Exponential and Logarithmic Functions
Exponential Functions

Given $b > 0$, investigate the behavior of the function $f(x) = b^x$ in two cases:

1. $0 < b < 1$
2. $b > 1$
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Plot of $f(x) = (1/3)^x$
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Exponential functions

We have

$$\lim_{x \to \infty} b^x = \begin{cases} \infty & \text{if } b > 1 \\ 1 & \text{if } b = 1 \\ 0 & \text{if } 0 < b < 1 \end{cases}$$

Equivalently

$$\lim_{x \to -\infty} b^x = \begin{cases} 0 & \text{if } b > 1 \\ 1 & \text{if } b = 1 \\ \infty & \text{if } 0 < b < 1 \end{cases}$$
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Exponential Functions

Some properties

\[ b^x + y = b^x b^y = b^{xy} = (b^x)^y = (b^y)^x \]

\[ b^0 = 1 \]

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The number $e$

$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$. 

$2 < e < 3$. 

Graph the function $(1 + \frac{1}{x})^x$ for $x$ large.
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Graph of \( \left(1 + \frac{1}{x}\right)^x \)
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Given two positive numbers $b$ and $z$, we define the logarithm of $z$ to the base $b$ to be the only number $s$ that solves the equation $z = b^s$.

Write $s = \log_b(z)$, or $z = b^{\log_b(z)}$.

Horizontal line test.
Given two positive numbers $b$ and $z$, we define the **logarithm of $z$ to the base $b$** to be the only number $s$ that solves the equation

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Logarithmic Functions: Graph of $e^x$
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- $\ln(x) = \log_{e}(x)$  Natural logarithm
Logarithmic Functions: Properties

\[ \log_b(xy) = \log_b(x) + \log_b(y) \]

\[ \log_b(x^y) = y \log_b(x) \]

\[ \log_b(1) = 0 \]

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Logarithmic Functions: Graph of $\ln(x)$
Example

The concentration of a drug in an organ seconds after it has been administered is given by

$$x(t) = 0.08 + 0.12 e^{-0.02 t},$$

where $x(t)$ is measured in grams per cubic centimeter.

(a) How long would it take for the concentration of the drug in the organ to reach 0.18 g/cm$^3$.

(b) How long would it take for the concentration of the drug in the organ to reach 0.16 g/cm$^3$.

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The height of a tree (in feet) of a certain kind of tree is approximated by

\[ h(t) = 160 + 240 e^{-0.2t}, \]

Estimate the age of an 80-ft tree.
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