10. General rules of probability

The Practice of Statistics in the Life Sciences
Second Edition

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Objectives (PSLS Chapter 10)

General rules of probability

- Independent events
- Conditional probability
- Multiplication rule
- Tree diagrams
- Diagnosis tests
Two events are **independent** if knowing that one event is true or has happened does not change the probability of the other event.

- “male” and “brown eyes” → independent
- “male” and “taller than 6 ft” → not independent
- “male” and “high cholesterol” → it’s not obvious (I’m guessing independent)
- “male” and “pregnant” → not independent
Sampling without replacement

*Pick one frog at random* from your target population *and don’t put it back*. Then pick another frog, etc.

Artificial pond with 10 male and 10 female frogs.

\[
P(1^{\text{st}} \text{ frog is male}) = \frac{10}{20} = 0.5.
\]

If 1\text{st} frog is male, \( P(2^{\text{nd}} \text{ frog is male}) = ? \)

¬ Here, successive picks are not independent.

Survey of a whole county with thousands of frogs (half males, half females).

\[
P(1^{\text{st}} \text{ frog is male}) = 0.5.
\]

If 1\text{st} frog is male, \( P(2^{\text{nd}} \text{ frog is male}) \approx ? \)

¬ Here, successive picks are “nearly” independent.
Conditional probability

Conditional probabilities reflect how the probability of an event can be different if we know that some other event has occurred or is true.

The conditional probability of event \( B \), given event \( A \) is:

\[
P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}
\]

(provided that \( P(A) \neq 0 \))

When two events \( A \) and \( B \) are independent, \( P(B \mid A) = P(B) \).

No information is gained from the knowledge of event \( A \).
Probabilities of hearing impairment and blue eyes among Dalmatian dogs.

\[ HI = \text{Dalmatian is hearing impaired} \]
\[ B = \text{Dalmatian is blue eyed} \]

\[ P(\text{HI and } B) = ? \quad P(\text{HI}) = ? \quad P(B) = ? \]
\[ P(\text{HI | } B) = \frac{P(\text{HI and } B)}{P(B)} = ? \]

Are the traits \textit{HI} and \textit{B} independent?
**General multiplication rule:**

The probability that any two events, \( A \) and \( B \), both occur is:

\[
P(A \text{ and } B) = P(A)P(B|A)
\]

**Multiplication rule for independent events:**

If \( A \) and \( B \) are independent, then:

\[
P(A \text{ and } B) = P(A)P(B)
\]
Artificial pond with 10 male and 10 female frogs
Successive captures are not independent.
Probability of randomly capturing 2 male frogs in a row:

\[ P(\text{male and male}) = P(1^{\text{st}} \text{ is male}) \times P(2^{\text{nd}} \text{ is male } | \text{ 1}^{\text{st}} \text{ is male}) \]

= ?

Blood donation center
Unrelated visitors are independent.
Probability that the next two unrelated visitors are both type O:

\[ P(O \text{ and } O) = ? \]
**Tree diagrams** are used to represent probabilities graphically and facilitate computations.

Probabilities of skin cancer among men and women by body locations.

In a random individual with skin cancer:

- $P(\text{head}) = ?$
- $P(\text{trunk}) = ?$
- $P(\text{limbs}) = ?$

% in each group who are women:

- $P(\text{woman} | \text{head}) = ?$
- $P(\text{woman} | \text{trunk}) = ?$
- $P(\text{woman} | \text{limbs}) = ?$
If a person gets a positive test result, what is the probability that he/she actually has the disease?

This is the positive predictive value: 

\[ \text{PPV} = P(\text{disease} \mid \text{positive test}) \]
**HIV-AIDS:** Suppose that about 1% of a large population has HIV-AIDS antibodies. The enzyme immuno-assay test has sensitivity .9985 and specificity .9940.

If a person who takes this test gets a positive test result, what is the probability that he or she actually has HIV-AIDS, \( P(\text{HIV-AIDS} \mid \text{positive test}) \)?
What’s the positive predictive value of the enzyme immuno-assay test for HIV-AIDS?

\[ PPV = P(\text{disease} \mid \text{positive}) \]

\[ P(\text{true positive}) = P(\text{disease and positive}) = ? \]

\[ P(\text{false positive}) = P(\text{no disease and positive}) = ? \]

\[ PPV = P(\text{disease} \mid \text{positive}) = ? \]