

# Graduate Algebra

## Homework 1

Fall 2014

Due 2014-09-03 at the beginning of class

1. Let  $S_n$  be the group of permutations of a set with  $n$  elements.
  - (a) Show that  $S_n$  has cardinality  $n!$ .
  - (b) Show that  $S_n$  is not abelian when  $n \geq 3$ .
2. Let  $H$  be a nonempty finite subset of a group  $G$ . Show that  $H$  is a subgroup if and only if  $ab \in H$  for every  $a, b \in H$ .
3. Let  $D_{2n}$  be the dihedral group with  $2n$ -elements.
  - (a) Show that  $D_6 \cong S_3$ .
  - (b) Show that  $D_{2n}$  is a proper subgroup of  $S_n$  for  $n \geq 4$ .
4. Let  $G$  be a finite semigroup and let  $T$  be the multiplication table for the binary operation.  $T$  is a matrix whose rows and columns are labeled by the elements of  $G$  and the entry on row  $g$  and column  $h$  is  $g \cdot h$ .
  - (a) Show that  $G$  is a group if and only if there is an element  $e$  such that the entry on row  $e$  and column  $g$  is  $g$ , the entry on column  $e$  and row  $g$  is  $g$ , and in each row and each column each element of  $G$  appears exactly once.
  - (b) Find all possible (non-isomorphic) groups of cardinality 1, 2, 3 and 4, by which I mean find all possible multiplication tables up to permutations of the labels of their rows and columns.