

# Math 30810 Honors Algebra 3

## Homework 1

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Due Thursday, September 1

Do any 4 of the following 5 questions. They're all about matrices. Artin refers to the textbook.

1. Artin 1.9 on page 32.
2. Artin 1.13 on page 32. [Hint: Do you know a good formula for  $\frac{1}{1+x}$  where  $x$  is a real number with  $|x| < 1$ ?]
3. Artin 6.2 on page 34.
4. Artin M.4 on page 35.
5. Suppose  $A$  and  $B$  are two  $n \times n$  matrices with complex entries and  $X$  is an indeterminate variable.
  - (a) If  $B$  is invertible show that  $\det(XI_n - AB) = \det(XI_n - BA)$  as degree  $n$  monic polynomials in  $X$ .
  - (b) Show that  $\det(XI_n - AB) = \det(XI_n - BA)$  even if  $B$  is not invertible. [Hint: Apply part (a) to  $B + aI_n$  for a suitable complex number  $a$ .]