

Math 30810 Honors Algebra 3

Homework 12

Andrei Jorza

Due at noon on Thursday, December 1

Do 8 of the following questions. Some questions are obligatory. Artin a.b.c means chapter a, section b, exercise c. You may use any problem to solve any other problem.

1. (You have to do this problem) Artin 11.6.8 (a), (b) on page 356.
2. (You have to do this problem) Let R be a ring and I an ideal of $R[X]$. For a polynomial $P(X) \in R[X]$ let $\ell(P)$ be the leading coefficient of $P(X)$. Define $J = \{\ell(P) \mid P \in I\}$. Show that J is an ideal of R . (This is very useful.)
3. Consider the ring $R = \mathbb{Z}[\sqrt{-14}] = \{m + n\sqrt{-14} \mid m, n \in \mathbb{Z}\}$. Let $I = (3, 1 + \sqrt{-14})$. Show that $I^2 = (9, 7 + \sqrt{-14})$ and that $I^4 = (5 + 2\sqrt{-14})$ and thus that I^4 is a principal ideal. (One can, in fact, show that the fourth power of any ideal in this ring is principal, but this would be the topic of a graduate number theory course.)
4. Artin 11.3.3 on page 354
5. Artin 11.3.4 on page 355.
6. Artin 11.3.9 on page 355.
7. Artin 11.3.10 on page 355.
8. Artin 11.4.4 on page 355.
9. Artin 11.6.7 on page 356.
10. Artin 11.M.7 on page 358.