Math 20550, Final Exam  
December 18, 2015

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- No calculators.
- The exam lasts for 2 hours.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 12 pages of the test.
- Each question is 7 points.
- You get 3 free points.

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

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PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!
Multiple Choice

1. (7 pts.) At what point does the surface parametrized by $\mathbf{r}(u, v) = (2u, u^2 + v^2, v + 1)$ have a tangent plane parallel to the plane $4x - 2y + 8z = 0$?

(a) (0, 0, 0)  (b) (2, 2, 2)  (c) (4, 8, 3)  (d) (4, -2, 8)  (e) (0, 0, 1)

2. (7 pts.) Compute $\int_{0}^{\sqrt{\pi/2}} \int_{y}^{\sqrt{\pi/2}} \cos(x^2) \, dx \, dy$. (Hint: change the order of integration.)

(a) -1  (b) 1/2  (c) $-\sqrt{\pi/2}$  (d) 0  (e) $\sqrt{\pi/2}$
3. (7 pts.) Find the tangential and normal components of the acceleration vector for the curve parametrized by \( \vec{r}(t) = (2 + t, t^2 - 2t, t^3) \) at the point \((2, 0, 0)\).

(a) \( a_N = \sqrt{5}, \ a_T = \sqrt{5} \)  
(b) \( a_N = 0, \ a_T = -4 \)

(c) \( a_N = 2/\sqrt{5}, \ a_T = -4/\sqrt{5} \)  
(d) \( a_N = 0, \ a_T = 0 \)

(e) \( a_N = 4, \ a_T = 0 \)

4. (7 pts.) Compute the flux of the vector field \( \vec{F} = x\vec{i} + y\vec{j} + z\vec{k} \) over the part of the cylinder \( x^2 + y^2 = 4 \) that lies between planes \( z = 0 \) and \( z = 2 \) with normal pointing away from the origin.

(a) \( 16\pi \)  
(b) \( 8\pi/3 \)  
(c) \( 8\pi \)  
(d) \( 24\pi \)  
(e) \( 0 \)
5.(7 pts.) Find the absolute maximum of \( f(x, y) = x^2 + 2y^2 + 4y - 2 \) on the disk \( x^2 + y^2 \leq 4 \).

(a) 18  (b) 0  (c) 6  (d) 16  (e) 14

6.(7 pts.) Find the work done by the force field \( \mathbf{F}(x, y) = yi + y\mathbf{j} \) moving a particle along the curve \( y = \sin x \) from the point \((0, 0)\) to the point \((\pi/2, 1)\).

(a) 3/2  (b) 1  (c) 0  (d) -3/2  (e) -1
7. (7 pts.) Use the Divergence theorem to find \( \iiint_S \mathbf{F} \cdot d\mathbf{S} \) where \( \mathbf{F}(x, y, z) = (xy, \frac{3}{4}y, -zy) \) and \( S \) is the closed surface \( x^2 + y^2 + z^2 = 4 \) with the outward orientation.

(a) \( 2\pi \)  (b) \( 16\pi \)  (c) \( \pi \)  (d) \( 0 \)  (e) \( 8\pi \)

8. (7 pts.) Let \( S \) be the portion of the graph \( z = 4 - 2x^2 - 3y^2 \) that lies over the region in the \( xy \)-plane bounded by \( x = 0, y = 0, \) and \( x + y = 1 \). Determine which of the following equals \( \iint_S (x^2 + y^2 + z) \, dS \).

(a) \( \int_0^1 \int_{1-x}^{1-y} (4 - 2x^2 - 3y^2) \sqrt{1 + 4x^2 + 9y^2} \, dx \, dy \)

(b) \( \int_0^1 \int_{0}^{1-x} \int_0^{4-2x^2-3y^2} x^2 + y^2 + z \, dz \, dy \, dx \)

(c) \( \int_0^1 \int_{0}^{1-x} (4 - x^2 - 2y^2) \sqrt{1 + 16x^2 + 36y^2} \, dx \, dy \)

(d) \( \int_0^1 \int_{0}^{1-y} \int_0^{4-2x^2-3y^2} 4 - x^2 - 2y^2 \, dz \, dx \, dy \)

(e) \( \int_0^1 \int_{0}^{x+y} 4 - x^2 - 2y^2 \, dy \, dx \)
9. (7 pts.) Find the area of the triangle with vertices \((2, 0, 0), (0, 1, 0)\) and \((0, 0, 1)\).

(a) \(\frac{3}{2}\) \hspace{1cm} (b) \(\sqrt{5}\) \hspace{1cm} (c) \(\frac{1}{2}\) \hspace{1cm} (d) 1 \hspace{1cm} (e) \(\frac{\sqrt{5}}{2}\)

10. (7 pts.) Find the maximum rate of change of \(f(x, y, z) = 3 \ln(x+y) + e^{yz}\) at the point \((1, 0, 2)\).

(a) \(\sqrt{20}\) \hspace{1cm} (b) 8 \hspace{1cm} (c) 5 \hspace{1cm} (d) \(\sqrt{34}\) \hspace{1cm} (e) 20
11. (7 pts.) Let \( f(x, y, z) = x^2z^3 - y^3z^2 \) and \( \mathbf{F} = \nabla f \) be its gradient vector field. Compute \( \int_C \mathbf{F} \cdot d\mathbf{r} \), where \( C \) is the curve \( \mathbf{r}(t) = (\cos(2\pi t), \sin(2\pi t), t) \), \( 0 \leq t \leq 1 \).

(a) 0  (b) \(-1\)  (c) \(2\pi + 1\)  (d) 1  (e) \(2\pi\)

12. (7 pts.) Let \( E \) be the part of the solid ball of radius 3, which lies below the cone \( z = \sqrt{x^2 + y^2} \) and above the plane \( z = 0 \). Suppose the density of \( E \) is given by \( \rho(x, y, z) = z^2 \). Which of the following integrals computes the mass of \( E \)?

(a) \( \int_0^{2\pi} \int_{\pi/4}^{\pi/2} \int_0^3 \rho^4 \sin \phi \cos^2 \phi \, d\rho \, d\phi \, d\theta \)
(b) \( \int_0^{\pi} \int_{\pi/2}^\pi \int_0^3 \rho^2 \sin \phi \cos \phi \, d\rho \, d\phi \, d\theta \)

(c) \( \int_0^{2\pi} \int_{\pi/4}^{\pi/2} \int_0^3 \rho^2 \cos^2 \phi \, d\rho \, d\phi \, d\theta \)
(d) \( \int_0^{2\pi} \int_0^{\pi/2} \int_0^3 \rho^4 \sin \phi \cos^2 \phi \, d\rho \, d\phi \, d\theta \)

(e) \( \int_0^{2\pi} \int_{\pi/4}^{\pi/2} \int_0^3 \rho^2 \cos^2 \phi \, d\rho \, d\phi \, d\theta \)
13. (7 pts.) Determine the length of the curve \( \mathbf{r}(t) = \langle t, 2\cos t, 2\sin t \rangle \) on the interval \( 0 \leq t \leq \pi \).

(a) \( 2\pi \)  (b) \( \sqrt{5}\pi \)  (c) \( 5\pi \)  (d) \( 3\pi \)  (e) \( 9\pi \)

14. (7 pts.) Find the equation for the tangent plane to the ellipsoid 
\( (x - 1)^2 + 2y^2 + (z + 2)^2 = 4 \) at the point \( (2, 1, -1) \).

(a) \(- (x - 2) + 2(y - 1) - 4(z + 1) = 0\)  (b) \((x - 2) + 2(y - 1) + 6(z + 1) = 0\)
(c) \(4(x - 2) + 4(y - 1) - 2(z + 1) = 0\)  (d) \(2(x - 2) + 4(y - 1) + 2(z + 1) = 0\)
(e) \(2(x - 2) + 4(y - 1) + 6(z + 1) = 0\)
15. (7 pts.) Let $S$ be the surface given by the graph of $z = x^2 + y^2$ over the rectangular region $D = [0, 1] \times [0, 1]$. Let $C$ be the boundary of $S$, oriented counterclockwise (when viewed from above). Use Stokes’ Theorem to evaluate $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = (e^{-x^2}, x + 3z, 3y)$.

(a) $\frac{2}{3}$  (b) $-\frac{2}{3}$  (c) 1  (d) 0  (e) $-1$

16. (7 pts.) Find the tangent line to the curve $\vec{r}(t) = (2t + 1, t^2 - 4, e^{1-t})$ at the point $(3, -3, 1)$

(a) $\langle x, y, z \rangle = (2, -6, 1)t + (3, -3, 1)$  (b) $\langle x, y, z \rangle = (2, 1, -e^{-1})t + (3, -3, 1)$
(c) $\langle x, y, z \rangle = (2, 2, -1)t + (3, -3, 1)$  (d) $\langle x, y, z \rangle = (2, 1, e^{-1})t + (3, -3, 1)$
(e) $\langle x, y, z \rangle = (2, 2, 1)t + (3, -3, 1)$
17. (7 pts.) Let $R$ be the parallelogram enclosed by the lines $x + 3y = 0$, $x + 3y = 2$, $x + y = 1$, and $x + y = 4$. Use the transformation $u = x + 3y$, $v = x + y$ to evaluate $\iint_R \frac{x + 3y}{(x + y)^2} \, dA$. (Hint: solving for $x$ and $y$ you get $x = \frac{1}{2}u + \frac{3}{2}v$, $y = \frac{1}{2}u - \frac{1}{2}v$.)

(a) $\frac{3}{4}$  
(b) 0  
(c) $\frac{3}{2}$  
(d) $-\frac{3}{4}$  
(e) $-\frac{3}{2}$

18. (7 pts.) Which one of the following integrals computes the volume of the part of the solid cylinder $x^2 + y^2 \leq 1$ that lies between planes $z = 0$ and $z = 2 - y$?

(a) $\int_0^{2\pi} \int_0^1 \int_0^2 (2 - r) \, r \, dz \, dr \, d\theta$  
(b) $\int_0^{2\pi} \int_0^1 \int_0^{2-r} r \, dz \, dr \, d\theta$

(c) $\int_0^{2\pi} \int_0^1 \int_0^{2-r \sin \theta} r \, dz \, dr \, d\theta$  
(d) $\int_0^{2\pi} \int_0^1 \int_0^2 r \, dz \, dr \, d\theta$

(e) $\int_0^{2\pi} \int_0^1 \int_0^2 (2 - r) \, dz \, dr \, d\theta$
19. (7 pts.) Use Green's Theorem to evaluate

\[ \int_C \left( -\frac{y^3}{3} + \sin(x) \right) \, dx + \left( \frac{x^3}{3} + y \right) \, dy \]

where \( C \) is the circle of radius 1 centered at \((0, 0)\) oriented counterclockwise when viewed from above.

(a) \( 4\pi \)  (b) \( 8\pi \)  (c) \( \pi \)  (d) \( 2\pi \)  (e) \( \frac{\pi}{2} \)

20. (7 pts.) Find \( \bar{r}(1) \), the position of the particle at time \( t = 1 \), if the acceleration at time \( t \) is \( \bar{a}(t) = (2t, 0, 3t^2) \), the initial velocity is \( \bar{v}(0) = (1, -1, 0) \) and initial position is \( \bar{r}(0) = (0, 0, 1) \).

(a) \( \left\langle \frac{4}{3}, -1, \frac{5}{4} \right\rangle \)  (b) \( \left\langle \frac{4}{3}, 1, \frac{5}{4} \right\rangle \)  (c) \( \left\langle -\frac{2}{3}, 1, \frac{5}{4} \right\rangle \)

(d) \( \left\langle \frac{4}{3}, -1, \frac{1}{4} \right\rangle \)  (e) \( \left\langle \frac{1}{3}, 0, \frac{5}{4} \right\rangle \)
21. (7 pts.) Use the second derivative test to classify critical points \((0, 0)\) and \((1, 2)\) of the function \(f(x, y) = 12x^2 + y^3 - 12xy.\)

(a) \((0, 0)\) is a local maximum and \((1, 2)\) is a local maximum.

(b) \((0, 0)\) is a saddle point and \((1, 2)\) is a local maximum.

(c) \((0, 0)\) is a local minimum and \((1, 2)\) is a local maximum.

(d) \((0, 0)\) is a saddle point and \((1, 2)\) is a local minimum.

(e) \((0, 0)\) is a local minimum and \((1, 2)\) is a local minimum.