Name: ____________________

Math 20550, Final Exam
December 18, 2015

Instructor: ANSWERS

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 2 hours.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 12 pages of the test.
- Each question is 7 points.
- You get 3 free points.

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(•)	(d)	(e)	11.	(a)	(b)	(c)	(•)	(e)
2.	(a)	(•)	(c)	(d)	(e)	12.	(•)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(•)	(d)	(e)	13.	(a)	(•)	(c)	(d)	(e)
4.	(•)	(b)	(c)	(d)	(e)	14.	(a)	(b)	(c)	(•)	(e)
5.	(a)	(b)	(c)	(d)	(•)	15.	(a)	(b)	(•)	(d)	(e)
6.	(•)	(b)	(c)	(d)	(e)	16.	(a)	(b)	(•)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(•)	17.	(•)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(•)	(d)	(e)	18.	(a)	(b)	(•)	(d)	(e)
9.	(•)	(b)	(c)	(d)	(e)	19.	(a)	(b)	(c)	(d)	(•)
10.	(a)	(b)	(c)	(•)	(e)	20.	(•)	(b)	(c)	(d)	(e)
						21.	(a)	(b)	(c)	(•)	(e)

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Multiple Choice

1.(7 pts.) At what point does the surface parametrized by $\vec{\mathbf{r}}(u, v) = \langle 2u, u^2 + v^2, v + 1\rangle$ have a tangent plane parallel to the plane $4x - 2y + 8z = 0$?

(a) $(0, 0, 0)$ (b) $(2, 2, 2)$ (c) $(4, 8, 3)$ (d) $(4, -2, 8)$ (e) $(0, 0, 1)$

2.(7 pts.) Compute $\int_0^{\sqrt{\pi/2}} \int_y^{\sqrt{\pi/2}} \cos(x^2)\, dx dy$. (Hint: change the order of integration.)

(a) -1 (b) $1/2$ (c) $-\sqrt{\pi/2}$ (d) 0 (e) $\sqrt{\pi/2}$

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3.(7 pts.) Find the tangential and normal components of the acceleration vector for the curve parametrized by $\vec{\mathbf{r}}(t) = \langle 2+t, t^2-2t, t^3\rangle$ at the point $(2,0,0)$.

(a) $a_N = \sqrt{5}$, $a_T = \sqrt{5}$

(b) $a_N = 0$, $a_T = -4$

(c) $a_N = 2/\sqrt{5}$, $a_T = -4/\sqrt{5}$

(d) $a_N = 0$, $a_T = 0$

(e) $a_N = 4$, $a_T = 0$

4.(7 pts.) Compute the flux of the vector field $\vec{\mathbf{F}} = x\vec{\mathbf{i}} + y\vec{\mathbf{j}} + z\vec{\mathbf{k}}$ over the part of the cylinder $x^2+y^2=4$ that lies between planes $z=0$ and $z=2$ with normal pointing away from the origin.

(a) 16π (b) $8\pi/3$ (c) 8π (d) 24π (e) 0

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5.(7 pts.) Find the absolute maximum of $f(x, y) = x^2+2y^2+4y-2$ on the disk $x^2+y^2 \leq 4$.

(a) 18 (b) 0 (c) 6 (d) 16 (e) 14

6.(7 pts.) Find the work done by the force field $\vec{\mathbf{F}}(x, y) = y\vec{\mathbf{i}} + y\vec{\mathbf{j}}$ moving a particle along the curve $y = \sin x$ from the point $(0, 0)$ to the point $(\pi/2, 1)$.

(a) $3/2$ (b) 1 (c) 0 (d) $-3/2$ (e) -1

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7.(7 pts.) Use the Divergence theorem to find $\iint_S \vec{\mathbf{F}} \cdot d\vec{\mathbf{S}}$ where $\vec{\mathbf{F}}(x, y, z) = \langle xy, \frac{3}{4}y, -zy \rangle$ and S is the closed surface $x^2 + y^2 + z^2 = 4$ with the outward orientation.

(a) 2π (b) 16π (c) π (d) 0 (e) 8π

8.(7 pts.) Let S be the portion of the graph $z = 4 - 2x^2 - 3y^2$ that lies over the region in the xy-plane bounded by $x = 0$, $y = 0$, and $x + y = 1$. Determine which of the following equals $\iint_S (x^2 + y^2 + z)\, dS$.

(a) $\int_0^1 \int_0^{1-y} (4 - 2x^2 - 3y^2)\sqrt{1 + 4x^2 + 9y^2}\, dx\, dy$

(b) $\int_0^1 \int_0^{1-x} \int_0^{4-2x^2-3y^2} x^2 + y^2 + z\, dz\, dy\, dx$

(c) $\int_0^1 \int_0^{1-x} (4 - x^2 - 2y^2)\sqrt{1 + 16x^2 + 36y^2}\, dy\, dx$

(d) $\int_0^1 \int_0^{1-y} \int_0^{4-2x^2-3y^2} 4 - x^2 - 2y^2\, dz\, dx\, dy$

(e) $\int_0^1 \int_0^{x+y} 4 - x^2 - 2y^2\, dy\, dx$

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9.(7 pts.) Find the area of the triangle with vertices $(2, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$.

(a) $\frac{3}{2}$ (b) $\sqrt{5}$ (c) $\frac{1}{2}$ (d) 1 (e) $\frac{\sqrt{5}}{2}$

10.(7 pts.) Find the maximum rate of change of $f(x, y, z) = 3\ln(x+y) + e^{yz}$ at the point (1, 0, 2).

(a) $\sqrt{20}$ (b) 8 (c) 5 (d) $\sqrt{34}$ (e) 20

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11.(7 pts.) Let $f(x,y,z) = x^2z^3 - y^3z^2$ and $\vec{\mathbf{F}} = \nabla f$ be its gradient vector field. Compute $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, where C is the curve $\vec{\mathbf{r}}(t) = \langle \cos(2\pi t), \sin(2\pi t), t \rangle$, $0 \le t \le 1$.

(a) 0 (b) -1 (c) $2\pi + 1$ (d) 1 (e) 2π

12.(7 pts.) Let E be the part of the solid ball of radius 3, which lies below the cone $z = \sqrt{x^2+y^2}$ and above the plane $z = 0$. Suppose the density of E is given by $\rho(x,y,z) = z^2$. Which of the following integrals computes the mass of E?

(a) $\int_0^{2\pi} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^3 \rho^4 \sin\phi \cos^2\phi \; d\rho \, d\phi \, d\theta$

(b) $\int_0^{\pi} \int_{\frac{\pi}{2}}^{\pi} \int_0^3 \rho^2 \sin\phi \cos\phi \; d\rho \, d\phi \, d\theta$

(c) $\int_0^{2\pi} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^3 \rho^2 \cos^2\phi \; d\rho \, d\phi \, d\theta$

(d) $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^3 \rho^4 \sin\phi \cos^2\phi \; d\rho \, d\phi \, d\theta$

(e) $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^3 \rho^2 \cos^2\phi \; d\rho \, d\phi \, d\theta$

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13.(7 pts.) Determine the length of the curve $\vec{\mathbf{r}}(t) = \langle t, 2\cos t, 2\sin t\rangle$ on the interval $0 \leq t \leq \pi$.

(a) 2π (b) $\sqrt{5}\pi$ (c) 5π (d) 3π (e) 9π

14.(7 pts.) Find the equation for the tangent plane to the ellipsoid $(x-1)^2 + 2y^2 + (z+2)^2 = 4$ at the point $(2, 1, -1)$.

(a) $-(x-2) + 2(y-1) - 4(z+1) = 0$ (b) $(x-2) + 2(y-1) + 6(z+1) = 0$

(c) $4(x-2) + 4(y-1) - 2(z+1) = 0$ (d) $2(x-2) + 4(y-1) + 2(z+1) = 0$

(e) $2(x-2) + 4(y-1) + 6(z+1) = 0$

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15.(7 pts.) Let S be the surface given by the graph of $z = x^2 + y^2$ over the rectangular region $D = [0,1] \times [0,1]$. Let C be the boundary of S, oriented counterclockwise (when viewed from above). Use Stokes' Theorem to evaluate $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, where $\vec{\mathbf{F}} = \langle e^{-x^2}, x + 3z, 3y \rangle$.

(a) $\frac{2}{3}$ (b) $-\frac{2}{3}$ (c) 1 (d) 0 (e) -1

16.(7 pts.) Find the tangent line to the curve $\vec{\mathbf{r}}(t) = \langle 2t+1, t^2-4, e^{1-t} \rangle$ at the point $(3, -3, 1)$

(a) $\langle x, y, z \rangle = \langle 2, -6, 1 \rangle t + \langle 3, -3, 1 \rangle$

(b) $\langle x, y, z \rangle = \langle 2, 1, -e^{-1} \rangle t + \langle 3, -3, 1 \rangle$

(c) $\langle x, y, z \rangle = \langle 2, 2, -1 \rangle t + \langle 3, -3, 1 \rangle$

(d) $\langle x, y, z \rangle = \langle 2, 1, e^{-1} \rangle t + \langle 3, -3, 1 \rangle$

(e) $\langle x, y, z \rangle = \langle 2, 2, 1 \rangle t + \langle 3, -3, 1 \rangle$

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17.(7 pts.) Let R be the parallelogram enclosed by the lines $x+3y=0$, $x+3y=2$, $x+y=1$, and $x+y=4$. Use the transformation $u=x+3y$, $v=x+y$ to evaluate $\iint_R \frac{x+3y}{(x+y)^2}\,dA$. (Hint: solving for x and y you get $x=-\frac{1}{2}u+\frac{3}{2}v$, $y=\frac{1}{2}u-\frac{1}{2}v$.)

(a) $\frac{3}{4}$ (b) 0 (c) $\frac{3}{2}$ (d) $-\frac{3}{4}$ (e) $-\frac{3}{2}$

18.(7 pts.) Which one of the following integrals computes the volume of the part of the solid cylinder $x^2+y^2\le 1$ that lies between planes $z=0$ and $z=2-y$?

(a) $\int_0^{2\pi}\int_0^1\int_0^2 (2-r)r\,dz\,dr\,d\theta$

(b) $\int_0^{2\pi}\int_0^1\int_0^{2-r} r\,dz\,dr\,d\theta$

(c) $\int_0^{2\pi}\int_0^1\int_0^{2-r\sin\theta} r\,dz\,dr\,d\theta$

(d) $\int_0^{2\pi}\int_0^1\int_0^2 r\,dz\,dr\,d\theta$

(e) $\int_0^{2\pi}\int_0^1\int_0^2 (2-r)\,dz\,dr\,d\theta$

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19.(7 pts.) Use Green's Theorem to evaluate

$$\int_C \left(-\frac{y^3}{3} + \sin(x)\right) dx + \left(\frac{x^3}{3} + y\right) dy$$

where C is the circle of radius 1 centered at $(0, 0)$ oriented counterclockwise when viewed from above.

(a) 4π (b) 8π (c) π (d) 2π (e) $\frac{\pi}{2}$

20.(7 pts.) Find $\vec{\mathbf{r}}(1)$, the position of the particle at time $t = 1$, if the acceleration at time t is $\vec{\mathbf{a}}(t) = \langle 2t, 0, 3t^2 \rangle$, the initial velocity is $\vec{\mathbf{v}}(0) = \langle 1, -1, 0 \rangle$ and initial position is $\vec{\mathbf{r}}(0) = \langle 0, 0, 1 \rangle$.

(a) $\left\langle \frac{4}{3}, -1, \frac{5}{4} \right\rangle$ (b) $\left\langle \frac{4}{3}, 1, \frac{5}{4} \right\rangle$ (c) $\left\langle -\frac{2}{3}, 1, \frac{5}{4} \right\rangle$

(d) $\left\langle \frac{4}{3}, -1, \frac{1}{4} \right\rangle$ (e) $\left\langle \frac{1}{3}, 0, \frac{5}{4} \right\rangle$

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21.(7 pts.) Use the second derivative test to classify critical points $(0, 0)$ and $(1, 2)$ of the function $f(x, y) = 12x^2 + y^3 - 12xy$.

(a) $(0, 0)$ is a local maximum and $(1, 2)$ is a local maximum.

(b) $(0, 0)$ is a saddle point and $(1, 2)$ is a local maximum.

(c) $(0, 0)$ is a local minimum and $(1, 2)$ is a local maximum.

(d) $(0, 0)$ is a saddle point and $(1, 2)$ is a local minimum.

(e) $(0, 0)$ is a local minimum and $(1, 2)$ is a local minimum.