Name: Practice Exam 1 - Multiple Choice
Instructor: Solution

Multiple Choice

1.(6 pts) Let $\mathbf{a} = \langle 1, 2, 0 \rangle$, $\mathbf{b} = \langle 3, 1, -1 \rangle$, and let $\mathbf{c} = \text{proj}_{\mathbf{a}}\mathbf{b}$ be the vector projection of $\mathbf{b}$ onto $\mathbf{a}$. Which one of the following vectors is orthogonal to $\mathbf{b} - \mathbf{c}$?

(a) $\langle 0, 1, 1 \rangle$ (b) $\langle 2, 1, -1 \rangle$ ~~(c)~~ $\langle 1, 2, 0 \rangle$

(d) $\langle 2, 1, 0 \rangle$ (e) $\langle 1, 0, 1 \rangle$

$\vec{c} = \text{proj}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \vec{a} = \frac{\langle 1,2,0 \rangle \cdot \langle 3,1,-1 \rangle}{1^2 + 2^2} \vec{a} = \frac{3+2}{5} \langle 1,2,0 \rangle = \langle 1,2,0 \rangle$

$\vec{b} - \vec{c} = \langle 3,1,-1 \rangle - \langle 1,2,0 \rangle = \langle 2,-1,-1 \rangle$

If $\vec{v}$ is a vector that is orthogonal to $\vec{b} - \vec{c}$, $\vec{v} \cdot (\vec{b} - \vec{c}) = 0$, or $\vec{v} \cdot \langle 2,-1,-1 \rangle = 0$. Examining the choices given, we see that the vector in $\boxed{(c)\ \langle 1,2,0 \rangle}$ is the answer because $\langle 1,2,0 \rangle \cdot \langle 2,-1,-1 \rangle = 0$

2.(6 pts) Find the radius of the sphere given by the equation

$$x^2 + y^2 + z^2 - 6x + 4z + 7 = 10.$$

(a) 3 (b) 9 (c) −4 (d) 2 ~~(e)~~ 4

$$x^2 + y^2 + z^2 - 6x + 4z + 7 = 10$$

$$\Leftrightarrow (x^2 - 6x + 3^2) - 3^2 + y^2 + (z^2 + 4z + 2^2) - 2^2 = 10 - 7$$

$$\Leftrightarrow (x-3)^2 + y^2 + (z+2)^2 = 3 + 9 + 4$$

$$\Leftrightarrow (x-3)^2 + y^2 + (z+2)^2 = \underbrace{16}_{=4^2}$$

So, the radius of the sphere is $\boxed{r = 4}$

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3.(6 pts) A particle moves with the position function $\mathbf{r}(t) = \langle t^2, -t, 2\rangle$. Find the normal component of acceleration.

~~(a)~~ $a_N = \dfrac{2}{\sqrt{1+4t^2}}$ (b) $a_N = 4t$ (c) $a_N = 2$

(d) $a_N = \dfrac{4t}{\sqrt{1+4t^2}}$ (e) $a_N = \sqrt{1+4t^2}$

$\vec{r}\,'(t) = \langle 2t, -1, 0\rangle$

$\vec{r}\,''(t) = \langle 2, 0, 0\rangle$

$|\vec{r}\,'(t)| = \sqrt{4t^2+1}$

$$\vec{r}\,'(t) \times \vec{r}\,''(t) = \begin{vmatrix} i & j & k \\ 2t & -1 & 0 \\ 2 & 0 & 0 \end{vmatrix} = 0\vec{i} - 0\vec{j} + (-2)\vec{k} = \langle 0, 0, -2\rangle$$

$$a_N = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|} = \frac{\sqrt{4}}{\sqrt{4t^2+1}} = \boxed{\frac{2}{\sqrt{4t^2+1}}}$$

4.(6 pts) Find the volume of the parallelepiped determined by the vectors $\mathbf{a} = \langle 1, 2, 2\rangle$, $\mathbf{b} = \langle 3, 2, 2\rangle$, and $\mathbf{c} = \langle 7, 3, 1\rangle$.

(a) -4 ~~(b)~~ 8 (c) 3 (d) -8 (e) 4

$V = |\vec{a} \cdot (\vec{b} \times \vec{c})|$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} 1 & 2 & 2 \\ 3 & 2 & 2 \\ 7 & 3 & 1 \end{vmatrix} \end{array} = 1(2-6) - 2(3-14) + 2(9-14)$$

$$= -4 - 2(-11) + 2(-5)$$

$$= -4 + 22 - 10 = 8$$

So, $\boxed{V = 8}$

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5.(6 pts) Where does the line with parametric equations

$$x = -1 + 3t \qquad y = 2 - 2t \qquad z = 3 + t$$

intersect the plane $3x + y - 4z = -4$?

(a) they do not intersect (b) $(-3, -3, -2)$ ~~(c)~~ $(8, -4, 6)$

(d) $(-10, 8, 0)$ (e) $(0, 0, 1)$

To find the point of intersection, we want to solve for t in this equation

$$\underbrace{3(-1+3t)}_{x} + \underbrace{(2-2t)}_{y} - 4\underbrace{(3+t)}_{z} = -4$$

$$\Rightarrow -3 + 9t + 2 - 2t - 12 - 4t = -4 \Rightarrow 3t = -4 + 3 - 2 + 12$$

$$\Rightarrow t = \frac{9}{3} \Rightarrow t = 3$$

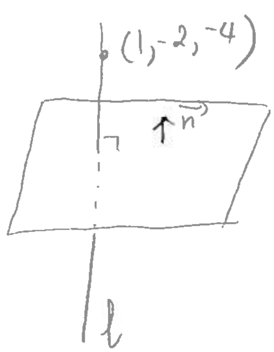
So, the pt of intersect is $\begin{cases} x = -1 + 3 \cdot 3 = 8 \\ y = 2 - 2 \cdot 3 = -4 \\ z = 3 + 3 = 6 \end{cases} \Rightarrow \boxed{(8, -4, 6)}$

6.(6 pts) Find symmetric equations for the line through the point $(1, -2, -4)$ which is orthogonal to the plane $2x - y + 3z = 18$.

(a) $\dfrac{x+1}{2} = \dfrac{y-2}{1} = \dfrac{z-4}{3}$ (b) $\dfrac{x-1}{2} = \dfrac{-y-2}{-1} = \dfrac{z+4}{3}$

~~(c)~~ $\dfrac{x-1}{2} = \dfrac{y+2}{-1} = \dfrac{z+4}{3}$ (d) $\dfrac{x-1}{\sqrt{14}} = \dfrac{-y-2}{\sqrt{14}} = \dfrac{z+4}{\sqrt{14}}$

(e) $\dfrac{1+2x}{\sqrt{14}} = \dfrac{-y-2}{\sqrt{14}} = \dfrac{-4+3z}{\sqrt{14}}$



the parallel vector of l is $\vec{n} = \langle 2, -1, 3 \rangle$

Symmetric eqs for l:

$$\frac{x-1}{2} = \frac{y-(-2)}{-1} = \frac{z-(-4)}{3}$$

$$\Rightarrow \boxed{\frac{x-1}{2} = \frac{y+2}{-1} = \frac{z+4}{3}}$$

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7.(6 pts) Find the position $\mathbf{r}(1)$ of a particle at time $\overset{t}{y} = 1$ if it has acceleration $\mathbf{a}(t) = e^t\mathbf{i} - 6t\mathbf{k}$, the initial position of the particle is $\mathbf{r}(0) = \langle 1, 0, -1\rangle$ and the initial velocity is $\mathbf{v}(0) = \langle 1, 1, 0\rangle$.

$\vec{a}(t) = \langle e^t, 0, -6t \rangle$

(a) $\mathbf{r}(1) = \langle 1, 0, 1\rangle$ (b) $\mathbf{r}(1) = \langle e, 0, 0\rangle$ (c) $\mathbf{r}(1) = \langle e, 1, -1\rangle$

~~(d)~~ $\mathbf{r}(1) = \langle e, 1, -2\rangle$ (e) $\mathbf{r}(1) = \langle 0, 1, 2\rangle$

$\vec{v}(t) = \int \vec{a}(t)\,dt = \langle e^t, 0, -3t^2 \rangle + \vec{C}$. $\vec{v}(0) = \langle 1, 0, 0\rangle + \vec{C} = \langle 1, 1, 0\rangle$

$\Rightarrow \vec{C} = \langle 0, 1, 0\rangle$

So, $\vec{v}(t) = \langle e^t, 1, -3t^2\rangle$. Now, $\vec{r}(t) = \int \vec{v}(t)\,dt = \langle e^t, t, -t^3\rangle + \vec{D}$

$\vec{r}(0) = \langle 1, 0, 0\rangle + \vec{D} = \langle 1, 0, -1\rangle \Rightarrow \vec{D} = \langle 0, 0, -1\rangle$

So, $\vec{r}(t) = \langle e^t, t, -t^3 - 1\rangle$. Hence, $\vec{r}(1) = \langle e^1, 1, -1^3 - 1\rangle$

$= \boxed{\langle e, 1, -2\rangle}$

8.(6 pts) Which of these is an equation of the tangent line to the curve

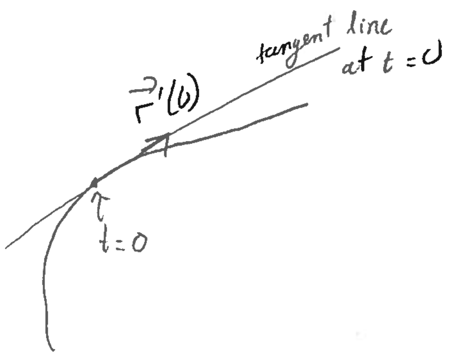
$$\mathbf{r}(t) = \langle t^2 + 2t + 3,\ 4t\cos(t),\ 2e^{3t}\rangle$$

at the point where $t = 0$?

(a) $\langle x, y, z\rangle = \langle 3, 4, 2e\rangle + t\langle 2, 0, 6e\rangle$ ~~(b)~~ $\langle x, y, z\rangle = \langle 3, 0, 2\rangle + t\langle 1, 2, 3\rangle$

(c) $\langle x, y, z\rangle = \langle 3, 0, 2\rangle + t\langle 1, -2, 3\rangle$ (d) $\langle x, y, z\rangle = \langle 3, 4, 2\rangle + t\langle 1, 2, 3\rangle$

(e) $\langle x, y, z\rangle = \langle 3, 0, 2e\rangle + t\langle 2, 4, 6\rangle$



$\vec{r}'(t) = \langle 2t + 2,\ 4\cos t - 4t\sin t,\ 6e^{3t}\rangle$

$\vec{r}'(0) = \langle 2, 4, 6\rangle$ (can take $\langle 1, 2, 3\rangle$ as parallel vector)

The pt (x, y, z) corresponds to $t = 0$ is

$$\begin{cases} x = 0^2 + 2\cdot 0 + 3 = 3 \\ y = 4\cdot 0 \cos 0 = 0 \\ z = 2e^{3\cdot 0} = 2 \end{cases}$$

So, $(x, y, z) = (3, 0, 2)$ is on the tangent line at $t = 0$

The eq of the tangent line at $t = 0$ is $\boxed{\langle x, y, z\rangle = \langle 3, 0, 2\rangle + t\langle 1, 2, 3\rangle}$

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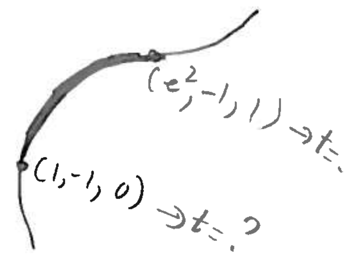
9.(6 pts) Which of the following expressions gives the length of the curve defined by $\mathbf{r}(t) = t^2\mathbf{i} - \mathbf{j} + \ln t\,\mathbf{k}$ between the points $(1,-1,0)$ and $(e^2,-1,1)$?

$\vec{r}(t) = \langle t^2, -1, \ln t \rangle$

(a) $\int_1^{e^2} \sqrt{4t^2 + 1/t^2}\,dt$ (b) $\int_1^{e} \sqrt{t^2 + 1 + \ln^2 t}\,dt$

(c) $\int_0^{1} \sqrt{2t + \ln t}\,dt$ (d) $\int_1^{e} \sqrt{2t + \ln t}\,dt$

~~(e)~~ $\int_1^{e} \sqrt{4t^2 + 1/t^2}\,dt$



$\vec{r}'(t) = \langle 2t, 0, \frac{1}{t} \rangle$.

$(1,-1,0) \Leftrightarrow t = 1$

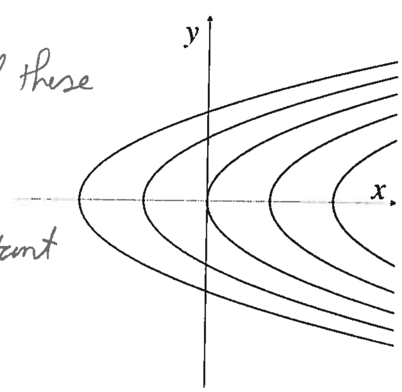
$(e^2,-1,1) \Leftrightarrow t = e$

$$L = \int_1^e |\vec{r}'(t)|\,dt = \int_1^e \sqrt{(2t)^2 + \left(\frac{1}{t}\right)^2}\,dt = \boxed{\int_1^e \sqrt{4t^2 + \frac{1}{t^2}}\,dt}$$

10.(6 pts) Which one of the following functions has level curves drawn below?

(a) $f(x,y) = y^2 + x$ (b) $f(x,y) = y + x^2$ (c) $f(x,y) = y - x^2$

~~(d)~~ $f(x,y) = y^2 - x$ (e) $f(x,y) = y^2 - x^2$



The equations of each of these curves is given by

$x = y^2 + k$, k is constant

$\Rightarrow x - y^2 = k$

or $y^2 - x = -k$. Or, we can just write $y^2 - x = k$ since k is any constant

So, (d) is the answer.