Multiple Choice

1. (6 pts) If the scalar projection of \( \mathbf{b} \) onto \( \mathbf{a} \) is \( \text{Comp}_a \mathbf{b} = 1 \), what is \( \text{Comp}_{2a} 3\mathbf{b} \)?

   (a) 2    (b) 5    (c) \( \frac{3}{2} \)    (d) 6    (e) 3

2. (6 pts) Which of the following expressions gives the length of the curve defined by
\[
\mathbf{r}(t) = \ln(t)\mathbf{i} - t\mathbf{j} + t^2\mathbf{k}
\]
between the points (0, -1, 1) and (1, -e, e^2)?

   (a) \( \int_1^e \sqrt{\ln^2(t) + t^2 + t^4} \, dt \)

   (b) \( \int_1^e \sqrt{1/t + 1 + 4t^2} \, dt \)

   (c) \( \int_1^e \sqrt{1/t^2 + 1 + 4t^2} \, dt \)

   (d) \( \int_1^e \sqrt{1/t - 1 + 2t} \, dt \)

   (e) \( \int_1^e \sqrt{\ln(t) - t + t^2} \, dt \)

3. (6 pts) Find the distance from the point (1, 2, 3) to the plane \( x + 2y - 2z = -7 \).

   (a) \( \sqrt{3} \)    (b) 1    (c) 6    (d) 2    (e) \( \sqrt{6} \)

4. (6 pts) Suppose the position function \( \mathbf{r}(t) = \langle t^3/3, t^2/2, t \rangle \). Find the normal component of the acceleration vector at \( t = 1 \).

   (a) \( a_N = \sqrt{2} \)    (b) \( a_N = \sqrt{3} \)    (c) \( a_N = \sqrt{5} \)

   (d) \( a_N = 1 \)    (e) \( a_N = 0 \)

5. (6 pts) Find the area of the triangle with vertices (4, 2, 2), (3, 3, 1) and (5, 5, 1).

   (a) 0    (b) 4    (c) \( \sqrt{3} \)    (d) \( \sqrt{6} \)    (e) 2
6. (6 pts) The two curves below intersect at the point \((1, 4, -1) = r_1(0) = r_2(1)\). Find the cosine of the angle of intersection

\[
\begin{align*}
r_1(t) &= e^{3t}i + 4\sin\left(t + \frac{\pi}{2}\right)j + (t^2 - 1)k \\
r_2(t) &= ti + 4j + (t^2 - 2)k
\end{align*}
\]

(a) \(\frac{1}{5}\)  \(\frac{1}{\sqrt{5}}\)  0  \(\frac{e}{\sqrt{e^2 + 4}}\)  3

7. (6 pts) Find the vector equation of the line passing through the point \((1, 1, 1)\) and \((1, 2, 3)\)

(a) \(\langle x, y, z \rangle = \langle 1, 1, 1 \rangle + t\langle 0, 1, 2 \rangle\)  
(b) \(\langle x, y, z \rangle = \langle 0, 1, 2 \rangle + t\langle 1, 2, 3 \rangle\)  
(c) \(\langle x, y, z \rangle = \langle 1, 1, 1 \rangle + t\langle 0, 0, 1 \rangle\)  
(d) None of the above  
(e) \(\langle x, y, z \rangle = \langle 1, 1, 1 \rangle + t\langle 1, 2, 3 \rangle\)

8. (6 pts) Consider a helix curve

\[r(t) = \langle \cos t - 1, \sin t, t \rangle.\]

Find the equation of the osculating plane of the curve at the point \((0, 0, 0)\)

(a) \(x = 0\)  
(b) \(-y + z = 0\)  
(c) None of the above  
(d) \(y + z = 0\)  
(e) \(x + y + z = 0\)

9. (6 pts) Given a space curve

\[r(t) = \langle 2\cos t, e^t, t \rangle.\]
Which of the following points is in the tangent line of the curve at the point $(2, 1, 0)$?

(a) $(1, 1, 1)$  (b) $(2, 1, 1)$  (c) $(1, 2, 0)$
(d) $(2, 2, 1)$  (e) $(0, 1, 2)$

10. (6 pts) Which of the following functions has this contour map

11. (6 pts) Given three points $P(2, 0, 2)$, $Q(1, 1, 0)$ and $R(1, 2, 3)$.
(a) Find an equation of the plane through $P$, $Q$ and $R$.
(b) Find an equation of the line through the point $(1, 1, 1)$ perpendicular to the plane in part (a).
12. (6 pts) (a) Find an equation for the line of intersection of the planes \( x - 3y + 2z = 0 \) and \( 2x - 3y + z = 0 \).

(b) Does the line from part (a) intersect the line with equations \( x = 1 + t, \ y = 3 - t, \ z = 1 + t \)? If so, where do they intersect?
13. (6 pts) A particle has the acceleration
\[ a(t) = 2j + 6t k. \]
At the time \( t = 0 \), the particle’s position is at the origin and its velocity is \( \mathbf{v}(0) = \mathbf{i} + \mathbf{k} \).
Find the position function \( \mathbf{r}(t) \) of the particle.
The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 6 pages of the test.
- Each multiple choice question is 6 points, each partial credit problem is 12 points. You will receive 4 extra points.

### Question 1
- (a) (b) (c) (d) (●)

### Question 2
- (a) (b) (●) (d) (e)

### Question 3
- (a) (b) (c) (●) (e)

### Question 4
- (●) (b) (c) (d) (e)

### Question 5
- (a) (b) (c) (●) (e)

### Question 6
- (a) (●) (c) (d) (e)

### Question 7
- (●) (b) (c) (d) (e)

### Question 8
- (a) (●) (c) (d) (e)

### Question 9
- (a) (b) (c) (●) (e)

### Question 10
- (a) (b) (●) (d) (e)

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**Please do NOT write in this box.**

### Multiple Choice

11.

12.

13.

**Extra Points.** 4

**Total**
11.

\[ \mathbf{PQ} = (2,0,2) - (1,1,0) = \langle 1, -1, 2 \rangle \]
\[ \mathbf{RQ} = (1,2,3) - (1,1,0) = \langle 0, 1, 3 \rangle \]

So for part (a) we compute \( \mathbf{PQ} \times \mathbf{RQ} \) which is \( \langle -3 - 2, -(3 - 0), 1 \rangle = \langle -5, -3, 1 \rangle \). So the equation of the plane through \( P, Q, \) and \( R \) is
\[ \langle -5, -3, 1 \rangle \cdot \langle x, y, z \rangle = \langle -5, -3, 1 \rangle \cdot \langle 1, 1, 0 \rangle \]
\[ -5x - 3y + z = -8. \]

For part (b) we use the computations from part (a) to see that the equation of the line should be
\[ \langle x, y, z \rangle = \langle 1, 1, 1 \rangle + t\langle -5, -3, 1 \rangle. \]

12. For part (a) we take the cross product of the normal vectors of each plane.
\[ \langle 1, -3, 2 \rangle \times \langle 2, -3, 1 \rangle = \langle -3 + 6, -(1 - 4), -3 + 6 \rangle = \langle 3, 3, 3 \rangle. \]

This is the direction of the line. So now we just need a point in the intersection of both planes, since both are of the form \( ax + by + cz = 0 \) we can easily see that \( (0,0,0) \) is in the intersection. So the equation for the line of intersection is
\[ \langle x, y, z \rangle = t\langle 3, 3, 3 \rangle. \]

For part (b) to see if this line intersects the given line, we must solve a system of equations. Changing the parameter in the given line to \( s \) we get
\[ 1 + s = 3t \]
\[ 3 - s = 3t \]
\[ 1 + s = 3t \]

Solving the first equation for \( s \) to get \( s = 3t - 1 \) and plugging into the 2nd equation to get \( 3 - (3t - 1) = 3t \) we can solve to see \( t = 2/3 \). Which would mean that \( s = 1 \). Checking these values in the equation we see that this is a solution \( 1 + 2 = 3(2/3) \). So using either the \( t = 2/3 \) or \( s = 1 \) value we can compute that the intersection point is \( (2,2,2) \).

13. To find \( \mathbf{v}(t) \) we integrate \( \mathbf{a}(t) \), to get
\[ \mathbf{v}(t) = \langle c, 2t + d, 3t^2 + e \rangle \]
where \( c, d, \) and \( e \) are constants.

To find the constants we evaluate at 0. To see that \( \mathbf{v}(0) = \langle c, d, e \rangle \) which should equal \( \langle 1, 0, 1 \rangle \). So \( c = 1, d = 0, \) and \( e = 1 \).

So now \( \mathbf{v}(t) = \langle 1, 2t, 3t^2 + 1 \rangle \).

To find \( \mathbf{r}(t) \) we repeat this process,
\[ r(t) = (t + c, t^2 + d, t^3 + t + e). \]

Now using the initial data we have that

\[ (0, 0, 0) = r(0) = (c, d, e). \]

So \( c = 0, \ d = 0, \) and \( e = 0. \)

So the final answer is \( r(t) = (t, t^2, t^3 + t). \)