Multiple Choice

1.(6 pts) Find the absolute maximum and minimum of $f(x, y) = 4y + x^2 - 2x + 1$ on the closed triangular region with vertices $(0, 0)$, $(2, 0)$ and $(0, 2)$.

(a) maximum value = 9, minimum value = 0

(b) maximum value = 10, minimum value = −1

(c) maximum value = 8, minimum value = 1

(d) maximum value = 4, minimum value = 0

(e) maximum value = 1, minimum value = 0

2.(6 pts) Find the equation of the tangent plane to the surface $xz + \ln(2x + y) = 5$ at the point $(-1, 3, -5)$.

(a) $-3x + y - z - 11 = 0$

(b) $4x - y + z + 12 = 0$

(c) $5x - y + z + 13 = 0$

(d) $3x + y - z - 5 = 0$

(e) $-4x + y - z - 4 = 0$

3.(6 pts) If $z = f(x, y)$, where f is differentiable, and $x = g(t), y = h(t)$, $g(1) = 3$, $h(1) = 4$, $g'(1) = -2$, $h'(1) = 5$, $f_x(3, 4) = 7$ and $f_y(3, 4) = 6$. Find dz/dt when $t = 1$.

(a) 16 (b) 23 (c) 44 (d) 32 (e) 13

4.(6 pts) Find the directional derivative of the function $f(x, y) = x^2 + y^3$ at the point $(2, 1)$ in the direction $< 1, 1 >$

(a) $\frac{7}{\sqrt{2}}$

(b) 7

(c) $\frac{3}{\sqrt{2}}$

(d) 3

(e) None of the above

5.(6 pts) For a function $f(x, y)$, suppose that $f_{xx} = x^2$ and $D(x, y) = f_{xx}f_{yy} - f_{xy}^2 = x^2y^2 - 2$. Which is true for the points $P(1, 1)$ and $Q(1, 2)$ where P and Q are critical points of f.

(a) P is a saddle point and Q is a local min.

(b) P is a saddle point and Q is a local max.

(c) P is a local min and Q is a local max.

(d) P is a local max and Q is a local min.

(e) None of the above

6.(6 pts) What is the equation of the tangent line to the curve of intersection between the two surfaces defined by $z = x^2 + y^2$ and $x^2 + 2y^2 + z^2 = 7$ at the point $(-1, 1, 2)$.

(a) $\langle x, y, z\rangle = \langle -1, 1, 2\rangle + t\langle 12, 10, -4\rangle$

(b) $\langle x, y, z\rangle = \langle -1, 1, 2\rangle + t\langle -2, 2, 1\rangle$

(c) $\langle x, y, z\rangle = \langle -1, 1, 2\rangle + t\langle -2, 4, 4\rangle$

(d) $\langle x, y, z\rangle = \langle -1, 1, 2\rangle + t\langle 1, 2, 1\rangle$

(e) None of the above

7.(6 pts) Find the maximum rate of change of $f(x, y) = 3e^{xy}$ at the point $(2, 0)$ and the direction in which it occurs.

(a) Rate of change $= 6$ in the direction $\langle 0, 1\rangle$

(b) Rate of change $= 3$ in the direction $\langle 1, 1\rangle$

(c) Rate of change $= \sqrt{3}$ in the direction $\langle 1, 0\rangle$

(d) Rate of change $= \sqrt{6}$ in the direction $\langle 1, -1\rangle$

(e) Rate of change $= 36$ in the direction $\langle -1, 0\rangle$

8.(6 pts) Find absolute maximum and minimum of $3x - y - 3z$ subject to the constraints $x + y - z = 0$ and $x^2 + 2z^2 = 6$.

(a) Max=12, Min=-12 (b) Max=15, Min=5 (c) Max=$3\sqrt{5}$, Min=0

(d) Max=5, Min=$-3\sqrt{5}$ (e) Max=6, Min=-1

9.(6 pts) Evaluate the iterated integral

$$\int_0^2 \int_y^{2y} 2xy\, dx\, dy.$$

(a) 12 (b) 3 (c) 4 (d) 5 (e) 2

10.(6 pts) Which integral represents the volume of the solid below the plane $x+y+z = 3$ and over the rectangle $[0, 2] \times [0, 1]$.

(a) $\int_0^2 \int_0^1 3 - x - y\, dydx$ (b) $\int_0^1 \int_0^2 3 - x - y\, dydx$

(c) $\int_0^2 \int_0^1 x + y + z\, dydx$ (d) $\int_0^1 \int_0^2 x + y + z\, dydx$

(e) $\int_0^2 \int_0^1 1\, dydx$

Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(12 pts) Find all critical points of of $f(x, y) = x^3 - xy + y^2/2$ and classify them using the second derivative test.

12.(12 pts) Use Lagrange Multipliers to find extrema values of the funciton $f(x, y) = 2x^3 - y^3$ subject to the contraint $x^2 + y^2 = 5$.

13.(12 pts) Find the volume of the solid that lies under the graph of $f(x, y) = xe^{xy}$ and above the rectangle $R = \{(x, y) \,|\, 0 \leq x \leq 1, 0 \leq y \leq 1\}$.

11. First we must find critical points, so we want to solve the system of equations:

$$f_x = 3x^2 - y = 0$$
$$f_y = -x + y = 0.$$

We get that $y = 3x^2$ from the first equation and then plugging into the 2nd equation we get $-x + 3x^2 = 0$ and so $x = 0$ or $-1 + 3x = 0$ so $x = 0$ or $x = 1/3$.

When $x = 0$, we get that $y = 0$ so we get the critical point $(0, 0)$. When $x = 1/3$ we get that $y = 1/3$ so we get the critical point $(1/3, 1/3)$.

Now we apply the 2nd derviative test, $f_{xx} = 6x$, $f_{xy} = -1$, $f_{yy} = 1$. So $D = 6x-1$. At $(0, 0)$, $D = -1$ so $(0, 0)$ is a saddle point. At $(1/3, 1/3)$, $D = 2$ and $f_{xx} = 3$ so $(1/3, 1/3)$ is a local minimum.

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12. The Lagrange system of equations is :

$$6x^2 = 2x\lambda$$
$$-3y^2 = 2y\lambda$$
$$x^2 + y^2 = 5$$

Case 1: Assume $x \neq 0$ and $y \neq 0$. Then we can divide the first two equations by $2x$ and y respectively, to get $3x = \lambda$ and $-3y = 2\lambda$. Solving for x and y in terms of λ we get $x = \lambda/3$ and $y = 2\lambda/-3$. Plugging these into the third equation we get that $\lambda^2/9 + 4\lambda^2/9 = 5$ which simplifies to $5\lambda^2/9 = 5$ and so $\lambda^2 = 9$ or $\lambda = \pm 3$.

So when $\lambda = 3$ we get $x = 1$, and $y = -2$ so we must check the point $(1, -2)$.

When $\lambda = -3$ we get $x = -1$ and $y = 2$ and so we must check the point $(-1, 2)$.

Case 2: Assume $x = 0$. Then the first equation will be satisfied for any λ value. And the third equation gives us that $y = \pm\sqrt{5}$. (The 2nd equation will give us that $\lambda = -15/2\sqrt{5}$ but this is irrelavant.). So we must check the points $(0, \sqrt{5})$ and $(0, -\sqrt{5})$.

Case 3: Assume $y = 0$ as in case 2 we will get that $x = \pm\sqrt{5}$ from equation 3, so we need to check the points $(\sqrt{5}, 0)$ and $(-sqrt5, 0)$.

Now we check all the points found in each of the cases:

$$\begin{aligned} f(1, -2) &= 2 - (-8) & &= 10 \\ f(-1, 2) &= -2 - 8 & &= -10 \\ f(0, \sqrt{5}) &= 0 - 5\sqrt{5} & &= -5\sqrt{5} \\ f(0, -\sqrt{5}) &= 0 - (-5\sqrt{5} & &= 5\sqrt{5} \\ f(\sqrt{5}, 0) &= 10\sqrt{5} - 0 & &= 10\sqrt{5} \\ f(-\sqrt{5}, 0) &= -10\sqrt{5} - 0 & &= -10\sqrt{5} \end{aligned}$$

So the maximum value is $10\sqrt{5}$ and the minimum value is $-10\sqrt{5}$.

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13. We want to compute the integral $\int_0^1 \int_0^1 xe^{xy}\, dydx$ (as integrating with respect to x first will be less desirable).

So we get

$$\int_0^1 e^{xy}|_0^1 \, dx = \int_0^1 e^x - 1 \, dx = e^x - x|_0^1 = e - 1 - (1 - 0) = e - 2$$