Multiple Choice

1.(6 pts)Which of the following integrals computes $\bar{x}$ for the solid bounded by $x = 0$, $y = 0$, $z = 0$, and $2x + 2y + z = 2$ which has constant density $\rho(x, y, z) = k$, and mass equals $k/3$.

(a) $1/3 \int_0^1 \int_0^1 \int_0^{1-x-y} x \, dzdydx$

(b) $3 \int_0^1 \int_0^{1-x} \int_0^{2-2x-2y} \, dxdydz$

(c) $3 \int_0^1 \int_0^{1-x} \int_0^{2-2x-2y} x \, dzdydx$

(d) $3 \int_0^1 \int_0^1 \int_0^2 x \, dxdydx$

(e) $1/3 \int_0^1 \int_0^1 \int_0^{1-x-y} \, dxdydz$

2.(6 pts) Express the double integral $\iint_D (x+1) \, dA$, where D is the region in the upper half-plane (i.e. $y \geq 0$) between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$, in polar coordinates.

(a) $\int_0^{2\pi} \int_2^3 (r^2 \cos\theta + r) \, drd\theta$

(b) $\int_0^{\pi} \int_2^3 (r \cos\theta + 1) \, drd\theta$

(c) $\int_0^{2\pi} \int_2^3 (r \cos\theta + 1) \, drd\theta$

(d) $\int_0^{\pi} \int_2^3 (r^2 \cos\theta + r) \, drd\theta$

(e) $\int_0^{\pi} \int_4^9 (r^2 \cos\theta + r) \, drd\theta$

3.(6 pts) Find the surface area of the parametric surface $\mathbf{r}(u, v) = \langle u, uv, u \rangle$ with $0 \leq u \leq 1, 0 \leq v \leq 1$.

(a) $\frac{1}{2}$ (b) $\sqrt{2}$ (c) 2 (d) 1 (e) $\frac{\sqrt{2}}{2}$

4.(6 pts) Which of the following integrals computes $\iiint_E y \, dV$ in cylindrical coordinates, where E is the solid that lies between cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$, above the xy-plane and below the plane $z = x + 4$?

(a) $\int_0^{2\pi}\int_1^2\int_0^{r\cos\theta+4} r^2\sin\theta\, dzdrd\theta$ (b) $\int_0^{2\pi}\int_1^2\int_0^{r\cos\theta+4} r\sin\theta\, dz\, dr\, d\theta$

(c) $\int_0^{2\pi}\int_1^2\int_0^{r\sin\theta}(r\cos\theta+4)\, dzdrd\theta$ (d) $\int_0^{2\pi}\int_1^2\int_0^{r} r^2\sin\theta\, dzdrd\theta$

(e) $\int_0^{2\pi}\int_1^2\int_0^{r} r\sin\theta\, dzdrd\theta$

5.(6 pts) Evaluate $\iiint_E (x^2+y^2+z^2)^{3/2} dV$, where E is the solid hemisphere enclosed by $x^2+y^2+z^2=1$ and above the plane $z=0$.

(a) 0 (b) $\frac{\pi}{3}$ (c) $\frac{1}{3}$ (d) $\frac{2\pi}{3}$ (e) π

6.(6 pts) Evaluate the line integral $\int_C (x+y+z)\, ds$ along the curve C given by $\mathbf{r}(t)=\langle \sin t, \cos t, t\rangle$, $0\le t\le \pi$.

(a) $\sqrt{2}\pi^2$ (b) $\frac{1}{2}\pi^2$ (c) $\sqrt{2}(2+\frac{1}{2}\pi^2)$

(d) $\frac{\sqrt{2}}{2}\pi^2$ (e) $2+\frac{1}{2}\pi^2$

7.(6 pts) Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y)=xy\mathbf{i}+e^x\mathbf{j}$ and C is the line segment from $(2,0)$ to $(4,0)$.

(a) 0 (b) 2 (c) -2 (d) 4 (e) -4

8.(6 pts) Use Fundamental Theorem of Line Integrals to compute $\int_C \nabla f\cdot d\mathbf{r}$ where $f(x,y,z)=xy^2+ye^{5z}$ and C is the curve $\mathbf{r}(t)=\langle e^{3t}, \sqrt{1+3t^4}, 2\sin(\pi t)\rangle$, $0\le t\le 1$.

(a) $4e^3$ (b) $4e^5$ (c) $2e^3$ (d) 0 (e) $2e^3+2$

9.(6 pts)Evaluate $\oint_C (y^3+x^2)dx+(3y^2x+x)dy$ where C is the positively oriented boundary of the triangle with vertices $(0,0)$, $(0,4)$, and $(2,2)$

(a) -2 (b) 2 (c) -4 (d) 0 (e) 4

10.(6 pts)Find an equation for the tangent plane to the surface given by $\mathbf{r}(u,v) = \langle u, uv, u\rangle$ at the point $(1,0,1)$.

(a) $x+z=0$

(b) $-x+z=0$

(c) $x+y+z=0$

(d) $y=1$

(e) $x+z=2$

Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(12 pts.) (a) Find the Jacobian of the transformation

$$x = u^2 - v^2, \qquad y = uv.$$

(b)(Note this part is not related to part (a)) Use the transformation $x = u + \frac{v}{2}$, $y = \frac{v}{2}$ to compute $\iint_D 2dA$ where D is the region bounded by $x^2 - 2xy + 5y^2 = 1$.

12.(12 pts.) For the integral $\int_0^1 \int_{4y}^4 e^{x^2} \, dx \, dy$

(a) Sketch the region of integration.
(b) Reverse the order of integration.
(c) Evaluate the integral .

13.(12 pts.) Suppose the vector field $F = y\mathbf{i} + (x+z)\mathbf{j} + (y+2z)\mathbf{k}$ is conservative. Find a potential function of F.

Name: ______________________

Instructor: ANSWERS

Math 20550. Exam 3
April 21, 2015

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 minutes..
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 5 pages of the test.
- Each multiple choice question is 6 points, each partial credit problem is 12 points. You will receive 4 extra points.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(•)	(d)	(e)
2.	(a)	(b)	(c)	(•)	(e)
3.	(a)	(b)	(c)	(d)	(•)
4.	(•)	(b)	(c)	(d)	(e)
5.	(a)	(•)	(c)	(d)	(e)
6.	(a)	(b)	(•)	(d)	(e)
7.	(•)	(b)	(c)	(d)	(e)
8.	(•)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(•)
10.	(a)	(•)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	________
11.	________
12.	________
13.	________
Extra Points.	4
Total:	________

11.(a)

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 2u & -2v \\ v & u \end{vmatrix} = 2u^2 + 2v^2$$

(b) First we compute the Jacobian

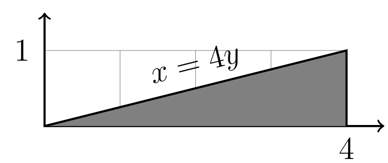
$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 1 & 1/2 \\ 0 & 1/2 \end{vmatrix} = 1/2$$

Then we see that our region D bounded by $x^2 - 2xy + 5y^2 = 1$ corresponds to the region S bounded by $u^2 + v^2 = 1$ under this transformation.

So our integral becomes

$$\iint_D 2dA = \iint_S 2(1/2)\, dA = \pi$$

12.(a) The limits of integration tell us that our region is bounded by $y = 0$, $y = 1$, $x = 4y$, and $x = 4$. This corresponds to the following picture



(b) $\displaystyle\int_0^1 \int_{4y}^4 e^{x^2}\, dx\, dy = \int_0^4 \int_0^{x/4} e^{x^2}\, dy\, dx$

(c) $\displaystyle\int_0^4 \int_0^{x/4} e^{x^2}\, dy\, dx = \int_0^4 ye^{x^2} \Big|_0^{x/4} dx = \frac{1}{4}\int_0^4 xe^{x^2}\, dx = \frac{1}{8}e^{x^2} \Big|_0^4 = \frac{1}{8}(e^{16} - 1)$

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13. First let's assume $f_x = y$, then

$$f = \int y\, dx = xy + c(y,z)$$

.

Now we want to use our candidate for f that we have found so far, to see if we can figure out the function $c(y,z)$ using the fact that we'd like $f_y = x + z$. We compute $f_y = x + c_y(y,z)$ using the f we found in the first step. Comparing $x + c_y(y,z)$ to $x + z$ we see that $c_y(y,z) = z$. So integrating we can find

$$c(y,z) = \int z\, dy = yz + d(z).$$

Now our candidate for f is

$$f = xy + yz + d(z).$$

To find $d(z)$ we compare this what we'd like to have for f_z which is $f_z = y + 2z$. Differentiating our candidate f with respect to z we get $f_z = y + d_z(z)$, and comparing to $y + 2z$ we see that $d_z(z) = 2z$. So to find $d(z)$ we integrate to get

$$d(z) = \int 2z\,dz = z^2 + a$$

where a can be any constant. For simplicity we choose a to be 0.

The final result is

$$f = xy + yz + z^2.$$