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Worksheet 7, Math 10560

1. Find the vector given by the projection of $v = \langle 2, 1, 5\rangle$ onto $a = \langle 1, -1, 2\rangle$.

   The formula is given by

   $$\frac{v \cdot a}{|a|^2} a$$

   Plugging in, we see that

   $$|a|^2 = 1 + 1 + 4 = 6$$

   and

   $$v \cdot a = 2 - 1 + 10 = 11$$

   Thus, the vector projection is

   $$\frac{11}{6}\langle 1, -1, 2\rangle$$

2. Does the following equation describe a sphere? If so, what is the orgin and the radius?

   $$2x^2 + 2y^2 + 2z^2 = 8x - 24z + 1$$

   We need to bring the equation into the form

   $$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$$

   We begin by completing the square on the left hand side, which gives

   $$2(x^2 - 4x + 4 - 4) + 2y^2 + 2(z^2 + 12z + 36 - 36) = 1$$

   which gives

   $$2(x-2)^2 + 2y^2 + 2(z+6)^2 = 1 + 8 + 72 = 81$$

   and hence

   $$(x-2)^2 + y^2 + (z-6)^2 = 81/2$$

   Thus the radius is $9/\sqrt{2}$ and the center is $(2, 0, -6)$.

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3. Determine the unit vector that has the same direction as $v = \langle 2, 4, -1 \rangle$.

   Being a unit vector means having length 1. So we need to multiply v by a positive real number so that it's length becomes 1. Indeed, $\frac{1}{|v|}v$ will do the job, since $|v| \neq 0$ and

$$\left| \frac{1}{|v|} |v| \right| = \frac{1}{|v|} |v| = 1.$$

   The length of v is

$$|v| = \sqrt{4 + 16 + 1} = \sqrt{21}$$

   so the unit vector pointing in the same direction as v is

$$\frac{1}{\sqrt{21}} \langle 2, 4, -1 \rangle$$

4. Find the area of the parallelpiped given by the vectors $v = \langle 1, 1, 1 \rangle$, $w = \langle 2, 1, 0 \rangle$ and $u = \langle 0, 2, 3 \rangle$.

   The volume of the parallellpiped is given by the value

$$|v \cdot (w \times u)|$$

   This amounts to computing the determinant

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 0 & 2 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix} - \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 0 & 2 \end{vmatrix} = 3 - 6 + 4 = 1$$

5. Let L be the line that contains the points $P(4, 2, -1)$ and $Q(-2, 5, 3)$. Does this line intersect the yz-plane? If so where?

A line is determined by a direction (i.e. a vector) and a point on that line. In this case the line L needs to pass through P and Q, so the direction should be given by the vector

$$\overrightarrow{PQ} = \langle -6, 3, 4 \rangle$$

So a parametric equation for the line L is then given by

$$P + tv = (4, 2, -1) + t(-6, 3, 4) = (4 - 6t, 2 + 3t, 4t - 1)$$

If this line is to pass through the yz-plane, then its x-component must be 0,

$$4 - 6t = 0$$

which means

$$t = 2/3$$

Plugging this in shows that the line L intersects the yz-plane at

$$(0, 4, 5/3)$$

6. Let v be a vector starting at the point $P(1, 3, 2)$ and such that

$$v \cdot \mathbf{i} = 2$$

$$v \cdot \mathbf{j} = 1$$

$$v \cdot \mathbf{k} = 4$$

Write down v in terms of its components. What is the terminal point of the vector v?

If a vector $w = \langle x, y, z \rangle$, then

$$w \cdot \mathbf{i} = x$$

$$w \cdot \mathbf{j} = y$$

$$w \cdot \mathbf{k} = z$$

so in particular

$$v = \langle 2, 1, 4 \rangle$$

Let Q denote the terminal point of v. Since we have

$$\overrightarrow{OP} + v = \overrightarrow{OQ}$$

we see that Q is given by

$$(1, 3, 2) + (2, 1, 4) = (3, 4, 6)$$