Math 20550 Calculus III Tutorial **Name:** ______________________________
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Tutorial Worksheet 2

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1. Let ℓ be the intersection of the planes given by equations $2x - 4y + z = 0$ and $3x - y - 2z + 9 = 0$. Find an equation for ℓ in the form $\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}$.

Solution: A point on both planes can be chosen as $(0, 1, 4)$. Now we compute

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & 1 \\ 3 & -1 & -2 \end{vmatrix} = \left\langle \begin{vmatrix} -4 & 1 \\ -1 & -2 \end{vmatrix}, -\begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix}, \begin{vmatrix} 2 & -4 \\ 3 & -1 \end{vmatrix} \right\rangle = \langle 9, 7, 10 \rangle$$

This gives us a vector parallel to ℓ and a point on ℓ, so an equation for it is $\mathbf{r}(t) = t\langle 9, 7, 10\rangle + \langle 0, 1, 4\rangle$.

2. A point moves in space in such a way that at time t its position is given by the vector-valued function $\mathbf{r}(t) = \left\langle t^2 + t + 1, -\frac{7}{2}t, -t + 4 \right\rangle$. At what time(s) does the point hit the plane $x + 2y + z = -5$?

Solution: We solve

$$t^2 + t + 1 + 2(-\frac{7}{2}t) + (-t + 4) = -5$$

and get $t = 2, 5$.

3. Determine the *speed* at $t = 2$ of an object whose position function is $\mathbf{r}(t) = \langle t^2 + 1, t^3, t^2 - 1 \rangle$.

Solution: $\mathbf{r}'(t) = \langle 2t, 3t^2, 2t \rangle$ so the speed is $|\,\mathbf{r}'(2)\,| = |\,\langle 4, 12, 4 \rangle\,| = \sqrt{16 + 144 + 16} = \sqrt{176}$.

4. Find an equation of the plane perpendicular to the line $x = 2016$, $y = 1 - 7t$, $z = 89 + 8t$ passing through the point $(1, 1, 1)$.

Solution: A normal vector of the plane is $\langle 0, -7, 8 \rangle$. Therefore an equation for the plane is given by

$$-7y + 8z = m.$$

Plug in $(1, 1, 1)$ and one sees $m = 1$.

5. The initial position and velocity of an object moving with acceleration $\mathbf{a}(t) = \mathbf{i} + 2\mathbf{j} + 6t\mathbf{k}$ are $\mathbf{r}(0) = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{v}(0) = \mathbf{j} - \mathbf{k}$. Find its position at time t.

Solution: $\mathbf{v}(t) = \int \mathbf{a}(t)\,dt = \int \langle 1, 2, 6t \rangle\ dt = \langle t, 2t, 3t^2 \rangle +\ \mathbf{c}$. Since $\mathbf{v}(0) = \langle 0, 1, -1 \rangle$ so $\mathbf{c} = \langle 0, 1, -1 \rangle$ and $\mathbf{v}(t) = \langle t, 2t + 1, 3t^2 - 1 \rangle$.

Then $\mathbf{r}(t) = \int \mathbf{v}(t)\,dt = \int \langle t, 2t + 1, 3t^2 - 1 \rangle\,dt = \left\langle \frac{t^2}{2}, t^2 + t, t^3 - t \right\rangle + \mathbf{c}$. Then $\mathbf{r}(0) = \langle 1, -2, 3 \rangle$ and $\mathbf{c} = \langle 1, -2, 3 \rangle$ and $\mathbf{r}(t) = \left\langle \frac{t^2}{2} + 1, t^2 + t - 2, t^3 - t + 3 \right\rangle$.

6. Find the distance from the point $(2, -2, 3)$ to the plane $6x + 4y - 3z = 2$.

Solution: We compute

$$|\frac{6 \cdot 2 + 4 \cdot (-2) - 3 \cdot 3 - 2}{\sqrt{6^2 + 4^2 + (-3)^2}}| = \frac{7}{\sqrt{61}}.$$

7. Find the acute angle of the intersection of the two curves $\mathbf{a}(u) = \langle 1 - u, u + 2, -3u \rangle$ and $\mathbf{b}(v) = \langle v, 2v, v^2 - v \rangle$.

Solution: First we find the point of the intersection by solving

$$1 - u = v$$

$$u + 2 = 2v$$

$$-3u = v^2 - v$$

This gives $u = 0$ and $v = 1$ and we see the intersection point is $(1, 2, 0)$. So $\mathbf{a}'(0) = \langle -1, 1, -3 \rangle$ and $\mathbf{b}'(1) = \langle 1, 2, 1 \rangle$. The cosine of the acute angle between these two vectors is $|\frac{\mathbf{a}'(0) \cdot \mathbf{b}'(1)}{|\mathbf{a}'(0)||\mathbf{b}'(1)|}| = \frac{2}{\sqrt{66}}$. So the angle is $\arccos(\frac{2}{\sqrt{66}})$.