Worksheet 7, Math 10560

1. Show that

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2 + y^2)} dA = \pi$$

and deduce that $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$.

2. Find the mass of the tetrahedron with points given by (0,0,0), (2,0,0), (0,3,0), (0,0,1) with density function $\rho(x, y, z) = x$.

3. Find the mass and center of mass of the solid E given by $z = 1 - y^2$ and the planes x + z = 1, x = 0, and z = 0 and constant density ρ .

4. Find the mass and center of mass of the solid S bounded by the paraboloid $z = 4x^2 + 4y^2$ and the plane z = a for a > 0, assuming that S has constant density K.