Math 20550 Calculus III Tutorial **Name:** ______________________________
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Tutorial Worksheet

Show all your work.

1. Evaluate (using spherical coordinates)

$$\iiint_E dV$$

where E is the solid that lies within $(x^2 + y^2 + z^2)^2 = 8z$.

2. Compute the volume of the solid defined by

$$x^2 + y^2 + z^2 - 2z \leq 0$$

and

$$x^2 + y^2 \leq \frac{3}{2}z.$$

(Use triple integrals in spherical coordinates. You can use the fact $\int \frac{\cos^3 x}{\sin^5 x} dx = -\frac{\cot^4 x}{4} + C$.)

3. Let the parallelogram D be defined by

$$5 \geq x + 2y \geq 2,$$

$$1 \geq y - x \geq -2.$$

Compute

$$\iint_D 2dA.$$

(Hint: Use change of variable: $u = x + 2y, v = y - x$. So $x = \frac{u-2v}{3}, y = \frac{u+v}{3}$.)

4. Let D be the region in the first quadrant that is defined by

$$1 \geq y^2 - x^2 \geq 0,$$

$$4 \geq xy \geq 3.$$

Use change of variable to compute the double integral

$$\iint_D (y^2 - x^2)^{xy}(x^2 + y^2)dA.$$

(Hint: let $u = y^2 - x^2, v = xy$. Using implicit differentiation we can obtain (try verifying one of them) $\frac{\partial x}{\partial u} = -\frac{x}{2(y^2+x^2)}, \frac{\partial y}{\partial u} = \frac{y}{2(y^2+x^2)}, \frac{\partial x}{\partial v} = \frac{y}{x^2+y^2}, \frac{\partial y}{\partial v} = \frac{x}{x^2+y^2}$.)