# Math 43900 Fall 2017 Problem Solving
# Lecture 1, August 22

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## The many stages of problem solving

1. I know this by heart. E.g., $(\ln x)' = \frac{1}{x}$.
2. I've seen problems exactly like this. E.g., $\int \frac{dx}{\sqrt{x^2 - x + 15}}$.
3. I've seen many time the methods necessary to do this problem. E.g., compute $(x^{x^x})'$.
4. Perhaps I can reinterpret/simplify the problem to something I've seen before. Questions: do I have to technical comfort to do this? Or the inventivity?
5. This seems like a geometry problem but it might really be something else.
6. What idea/trick/perspective could the author of the problem have been aiming for? Remember: this problem has been done by someone in a reasonable amount of time. It's not a research problem. This can be extremely helpful.
7. "How do I" questions. How do I choose a problem to work on? How do I brainstorm for (or build a repository of) helpful ideas? What do I do when I get stuck?
8. I don't even understand what the problem is saying.
9. Uncontrolled laughter at the absurdity and impossibility of the problem.

Some exercises.

1. Determine all prime numbers with $n$ with $k \geq 3$ digits (in base 10) with the following property: no matter how you eliminate at most $k-2$ digits from the decimal expansion of $n$, the resulting number is still prime.
2. For what positive integers $n$ is $\sqrt{n+3} + \sqrt{n + \sqrt{n+3}}$ an integer?
3. You know $xy = 6$ for reals $x, y > 0$. If $x, y > 2$ show that $x + y < 5$.
4. A convex polygon $A_1 A_2 \ldots A_n$ has vertices with integral coordinates and all the vertices lie on a circle. You know that the squares of the side lengths of the polygon are integers divisible by a fixed odd positive integer $n$. Show that twice the area of the polygon is an integer divisible by $n$.